Advanced Higher Derivatives

Note: nothing is given in the exam – remember everything !

In the following, a, b and n are real constants and $f(x)$ a real-valued function with no restrictions unless otherwise stated.

Sine and Cosine

$$\frac{d}{dx}\left(\sin f(x)\right)^n = n\, f'(x)\left(\sin f(x)\right)^{n-1}\cos f(x)$$

$$\frac{d}{dx}\left(\cos f(x)\right)^n = -\,n\, f'(x)\left(\cos f(x)\right)^{n-1}\sin f(x)$$

Special cases

- $n = 1$ gives

$$\frac{d}{dx}\left(\sin f(x)\right) = f'(x)\cos f(x)$$

$$\frac{d}{dx}\left(\cos f(x)\right) = -\,f'(x)\sin f(x)$$

Tangent

$$\frac{d}{dx}\left(\tan f(x)\right)^n = n\, f'(x)\left(\tan f(x)\right)^{n-1}\sec^2 f(x)$$

$$\left(f(x) \neq \frac{(2m+1)\pi}{2},\ m \in \mathbb{Z}\right)$$

Special cases

- $n = 1$ gives

$$\frac{d}{dx}\left(\tan f(x)\right) = f'(x)\ \sec^2 f(x)$$

- $f(x) = ax + b$ gives

$$\frac{d}{dx}\left(\tan(ax+b)\right)^n = n\ a\left(\tan(ax+b)\right)^{n-1}\sec^2(ax+b)$$

- $f(x) = ax + b$, $n = 1$ gives

$$\frac{d}{dx}\tan(ax+b) = a\ \sec^2(ax+b)$$

- $f(x) = x$, $n = 1$ gives

$$\frac{d}{dx}\tan x = \sec^2 x$$

Reciprocal Trigonometric Functions

$$\frac{d}{dx}\left(\sec f(x)\right)^n = n\ f'(x)\left(\sec f(x)\right)^{n-1}\sec f(x)\tan f(x)$$

$$\left(f(x) \neq \frac{(2m+1)\pi}{2}, m \in \mathbb{Z}\right)$$

$$\frac{d}{dx}\left(\operatorname{cosec} f(x)\right)^n = -\,n\ f'(x)\left(\operatorname{cosec} f(x)\right)^{n-1}\operatorname{cosec} f(x)\cot f(x)$$

$$\left(f(x) \neq m\pi, m \in \mathbb{Z}\right)$$

$$\frac{d}{dx}\left(\cot f(x)\right)^n = -\,n\,f'(x)\left(\cot f(x)\right)^{n-1}\operatorname{cosec}^2 f(x)$$

$$\left(f(x) \neq m\pi,\ m \in \mathbb{Z}\right)$$

Special cases

- $n = 1$ gives

$$\frac{d}{dx}\left(\sec f(x)\right) = f'(x)\ \sec f(x)\tan f(x)$$

$$\frac{d}{dx}\left(\operatorname{cosec} f(x)\right) = -\,f'(x)\ \operatorname{cosec} f(x)\cot f(x)$$

$$\frac{d}{dx}\left(\cot f(x)\right) = -\,f'(x)\operatorname{cosec}^2 f(x)$$

- $f(x) = ax + b$ gives

$$\frac{d}{dx}\left(\sec(ax+b)\right)^n = na\left(\sec(ax+b)\right)^{n-1}\sec(ax+b)\tan(ax+b)$$

$$\frac{d}{dx}\left(\operatorname{cosec}(ax+b)\right)^n = -\,na\left(\operatorname{cosec}(ax+b)\right)^{n-1}\operatorname{cosec}(ax+b)\cot(ax+b)$$

$$\frac{d}{dx}\left(\cot(ax+b)\right)^n = -\,na\left(\cot(ax+b)\right)^{n-1}\operatorname{cosec}^2(ax+b)$$

- $f(x) = ax + b$, $n = 1$ gives

$$\frac{d}{dx}\sec(ax+b) = a\sec(ax+b)\tan(ax+b)$$

$$\frac{d}{dx}\operatorname{cosec}(ax+b) = -\,a\operatorname{cosec}(ax+b)\cot(ax+b)$$

$$\frac{d}{dx}\cot(ax+b) = -a\,\text{cosec}^2(ax+b)$$

- $f(x) = x, n = 1$ gives

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\text{cosec}\, x = -\text{cosec}\, x \cot x$$

$$\frac{d}{dx}\cot x = -\text{cosec}^2 x$$

Inverse Trigonometric Functions

$$\frac{d}{dx}\left(\sin^{-1} f(x)\right)^n = \frac{n\, f'(x)\left(\sin^{-1} f(x)\right)^{n-1}}{\sqrt{1-\left(f(x)\right)^2}} \quad \left(\left|f(x)\right| < 1\right)$$

$$\frac{d}{dx}\left(\cos^{-1} f(x)\right)^n = -\frac{n\, f'(x)\left(\cos^{-1} f(x)\right)^{n-1}}{\sqrt{1-\left(f(x)\right)^2}} \quad \left(\left|f(x)\right| < 1\right)$$

$$\frac{d}{dx}\left(\tan^{-1} f(x)\right)^n = \frac{n\, f'(x)\left(\tan^{-1} f(x)\right)^{n-1}}{1+\left(f(x)\right)^2}$$

Special cases

- $n = 1$ gives

$$\frac{d}{dx} \sin^{-1} f(x) = \frac{f'(x)}{\sqrt{1 - \left(f(x)\right)^2}}$$

$$\frac{d}{dx} \cos^{-1} f(x) = -\frac{f'(x)}{\sqrt{1 - \left(f(x)\right)^2}}$$

$$\frac{d}{dx} \tan^{-1} f(x) = \frac{f'(x)}{1 + \left(f(x)\right)^2}$$

- $f(x) = \frac{x}{a}$ $(a \neq 0), n = 1$ gives

$$\frac{d}{dx} \sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} \cos^{-1}\left(\frac{x}{a}\right) = -\frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} \tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$$

- $f(x) = x, n = 1$ gives

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$$

Exponentials

$$\frac{d}{dx}\, e^{f(x)} = f'(x)\, e^{f(x)}$$

Special cases

- $f(x) = ax + b$ gives

$$\frac{d}{dx}\, e^{ax+b} = a\, e^{ax+b}$$

- $f(x) = x$ gives

$$\frac{d}{dx}\, e^{x} = e^{x}$$

Logarithms

$$\frac{d}{dx}\ \ln\, f(x) = \frac{f'(x)}{f(x)} \qquad (f(x) > 0)$$

Special cases

- $f(x) = ax + b$ gives

$$\frac{d}{dx}\ \ln\,(ax+b) = \frac{a}{ax+b}$$

- $f(x) = x$ gives

$$\frac{d}{dx}\ \ln\, x = \frac{1}{x}$$