C100/SQP321

Maths Higher

NATIONAL QUALIFICATIONS

Contents *Page*

SCOTTISH QUALIFICATIONS AUTHORITY

Course Assessment Specification

Mathematics Higher

The purpose of this document is to provide:

- details of the structure of the Question Papers in this Course
- guidance to centres on how to use information gathered from the Question Papers in this Course to estimate candidate performance.

Part 1

This part of the Course Assessment Specification details the structure of the Question Papers in this Course.

The Course assessment consists of two Question Papers.

Paper	Time Allocation	Mark Allocation
1	1 hour 30 mins	70
2	1 hour 10 mins	60

Paper 1

- In this paper no access to a calculator is allowed. All questions within this paper are classified as non-calculator.
- This paper has two sections, A and B.

Section A – total marks 40

- This section of the paper consists of Objective Test Questions.
- There are 20 questions, each worth 2 marks.
- At least 75% of these questions will provide opportunities at grade C.

Section B – total marks 30

- Questions in this section of the paper will be short and extended response.
- There will be approximately three to five questions.

Paper 2 – total marks 60

- In this paper, access to a calculator is allowed.
- Questions within this paper are classified as calculator required (CR) or calculator neutral (CN). In calculator neutral questions, there is no advantage to using a calculator.
- Questions in this the paper will be short and extended response.
- There will be approximately six to eight questions.

The Question Papers will assess the candidate's ability to retain and integrate mathematical knowledge across the Units of the Course.

The Question Papers are designed to fit the following criteria:

- there should be as wide a syllabus coverage as possible
- approximately 65% of the total marks will provide opportunities at Grade C
- at least one quarter of the marks should be allocated to **each** of the three Units
- at least one quarter of the marks should be defined under **each** of the headings – selecting a strategy, processing data, interpreting and communicating information.

Part 2

This part of the Course Assessment Specification provides guidance on how to use assessment information gathered from the Question Papers to estimate candidate performance.

The Course award is based on two Question Papers with a total mark of 130.

In National Qualifications, cut-off scores should be set at approximately 70% for Grade A and 50% for Grade C with Grade B falling midway.

For a total mark range of 0-130, the following gives an indication of the cut-off scores:

Grade	Band	Mark Range
A	1	111-130
A	2	91-110
B	3	85-90
B	4	78-84
C	5	72-77
C	6	65-71
D	7	58-64
NA	8	52-57
NA	9	45-51

These cut-off scores may be lowered if the Question Papers turn out to be more demanding, or raised if the Question Papers are less demanding.

Worked example

- In a centre's own prelim, a candidate scores 79/130.
- The prelim covers only two Units of the Course and the centre's view is that it is slightly less demanding than the SQA's examination.
- Using the mark range, a realistic estimate may be **band 5** rather than band 4.

[C100/SQP321]

Mathematics
Higher
Paper 1
Specimen Question Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

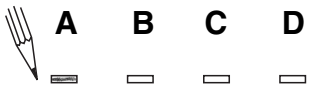
A 8

B 1

C 0

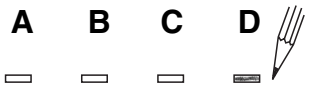
D −4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. If $y = \frac{x^3 - x}{x^2}$, what is $\frac{dy}{dx}$?

A $\frac{3x^2 - 1}{2x}$

B $1 + \frac{1}{x^2}$

C $\frac{3}{2}x - \frac{1}{2}$

D $x^3 - x - x^{-2}$

2. Functions f and g are given by $f(x) = 2x - 3$ and $g(x) = x^2$.
Find an expression for $g(f(x))$.

A $g(f(x)) = 4x^2 - 12x + 9$

B $g(f(x)) = x^2 + 2x - 3$

C $g(f(x)) = 4x - 9$

D $g(f(x)) = 2x^3 - 3x^2$

3. Find $\int \frac{1}{\sqrt[3]{x}}\, dx$.

A $-\frac{3}{2}x^{-\frac{1}{2}} + c$

B $x^{-3} + c$

C $\frac{3}{2}x^{\frac{2}{3}} + c$

D $-2x^{-2} + c$

4. A and B have coordinates (2, 3, –2) and (–1, –4, 0) respectively.

What is the distance between A and B?

A $\sqrt{6}$

B $\sqrt{17}$

C $\sqrt{62}$

D $\sqrt{148}$

5. A sequence is defined by the recurrence relation

$$u_{n+1} = 3u_n - 4,\ u_0 = -1.$$

What is the value of u_2?

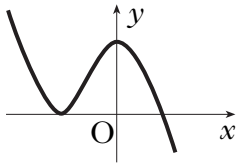
A –25

B –10

C –4

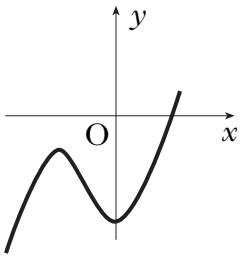
D –1

6. The diagram shows a sketch of $y = f(x)$.

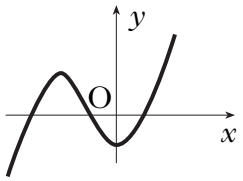


Which of the diagrams below shows a sketch of $y = -3 - f(x)$?

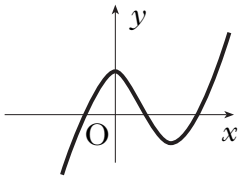
A



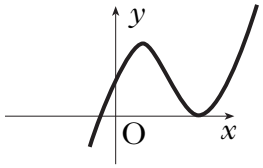
B



C



D



7. Which of the following describes the stationary point on the curve with equation $y = 3(x - 4)^2 - 5$?

A minimum at (4, 5)

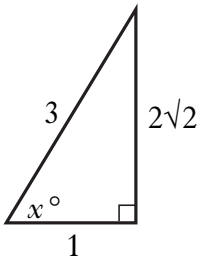
B maximum at (4, 5)

C minimum at (4, –5)

D maximum at (4, –5)

8. The diagram shows a right-angled triangle with sides of 1, $2\sqrt{2}$ and 3.

What is the value of $\sin 2x°$?



A $\frac{4\sqrt{2}}{9}$

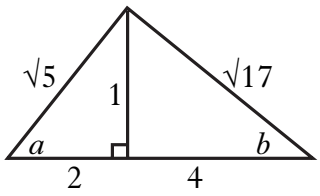
B $\sqrt{\frac{2}{3}}$

C $\frac{4\sqrt{2}}{3}$

D $\frac{3}{\sqrt{2}}$

9. a and b are angles as shown in the diagram.

What is the value of $\sin(a - b)$?



A $-\frac{7}{\sqrt{85}}$

B $\frac{2}{\sqrt{85}}$

C $\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{17}}$

D $\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{17}}$

10. A circle has equation $x^2 + y^2 + 8x - 6y - 12 = 0$.

What is the radius of this circle?

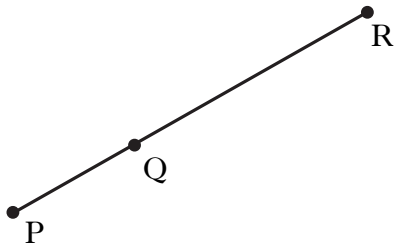
A $\sqrt{2}$

B $\sqrt{19}$

C $\sqrt{37}$

D $\sqrt{88}$

11. The points P(1, 3, 7), Q(5, 13, 13) and R(s, 33, 25) are collinear as shown in the diagram.



What is the value of s?

A 9

B 10

C 13

D 31

12. If $2x^2 - 12x + 11$ is expressed in the form $2(x - b)^2 + c$, what is the value of c?

A -25

B -7

C 11

D 23

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 3x^2 + 9x + 1$ and the curve passes through the origin.

What is the equation of the curve?

A $y = x^3 + \frac{9}{2}x^2 + x$

B $y = 6x^3 + 9x^2$

C $y = 3x^3 + 9x^2 + x + 1$

D $y = 6x + 9$

14. For what value of k does the equation $x^2 - 3x + k = 0$ have equal roots?

A $-\frac{9}{4}$

B $-\frac{1}{12}$

C 0

D $\frac{9}{4}$

15. The point P(–1, 2) lies on the circle with equation $x^2 + y^2 - 6x - 8y + 5 = 0$.
What is the gradient of the tangent at P?

A -2

B $-\frac{1}{3}$

C $\frac{6}{7}$

D $\frac{1}{2}$

16. What is the value of $\int_0^{\frac{\pi}{6}} 4\cos 2x\,dx$?

A -2

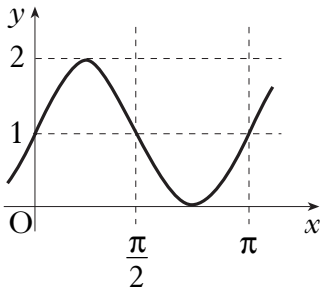
B $-\sqrt{\frac{3}{2}}$

C $\sqrt{3}$

D 4

17. The graph shown in the diagram has equation of the form $y = \sin(px) + q$.

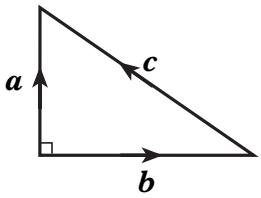
What are the values of p and q?



	p	q
A	2	1
B	$\frac{1}{2}$	1
C	2	2
D	$\frac{1}{2}$	2

18. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are represented by the sides of a right-angled triangle as shown in the diagram.

$|\boldsymbol{a}| = 3$ and $|\boldsymbol{c}| = 5$.



Here are two statements about these vectors:

(1) $\boldsymbol{a}.\boldsymbol{c} = 9$

(2) $\boldsymbol{a}.\boldsymbol{b} = -1$

Which of the following is true?

A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

19. If $\log_3 t = 2 + \log_3 5$, what is the value of t?

A 7

B 10

C 25

D 45

20. If $3^k = e^4$, find an expression for k.

A $k = \sqrt[3]{4^e}$

B $k = \dfrac{e^4}{3}$

C $k = 4 / \log_e 3$

D $k = 1 / \log_e 3$

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. A firm cleans the factory floor on a daily basis with disinfectant. It has a choice of two products, either "A" or "B".

Product A removes 70% of all germs but during the next 24 hours, 300 "new" germs per sq unit are estimated to appear.

Product B removes 80% of all germs but during the next 24 hours, 350 "new" germs per sq unit are estimated to appear.

For product A, let u_n represent the number of germs per sq unit on the floor immediately before disinfecting for the nth time.

For product B, let v_n represent the number of germs per sq unit on the floor immediately before disinfecting for the nth time.

(*a*) Write down a recurrence relation for each product to show the number of germs per sq unit present prior to disinfecting. **2**

(*b*) Determine which product is more effective in the long term. **4**

22. (*a*) Find the stationary points on the curve with equation $y = x^3 - 9x^2 + 24x - 20$ and justify their nature. **7**

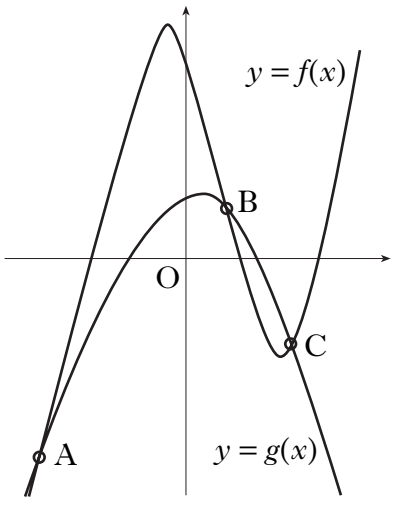
(*b*) (i) Show that $(x - 2)^2(x - 5) = x^3 - 9x^2 + 24x - 20$.

(ii) Hence sketch the graph of $y = x^3 - 9x^2 + 24x - 20$. **4**

23. The diagram shows a sketch of functions f and g where $f(x) = x^3 + 5x^2 - 36x + 32$ and $g(x) = -x^2 + x + 2$.

The two graphs intersect at the points A, B and C.

Determine the x-coordinate of each of these three points. **8**



24. Find the solution(s) of the equation $\sin^2 p - \sin p + 1 = \cos^2 p$ for $\frac{\pi}{2} < p < \pi$. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

[C100/SQP321]

Mathematics
Higher
Paper 2

Specimen Question Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Read Carefully

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SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

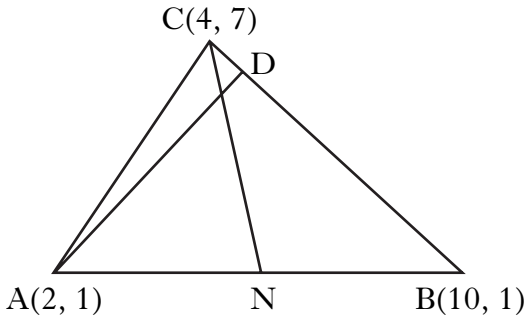
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Triangle ABC has coordinates A(2, 1), B(10, 1) and C(4, 7).



(*a*) Find the equation of the median CN. **3**

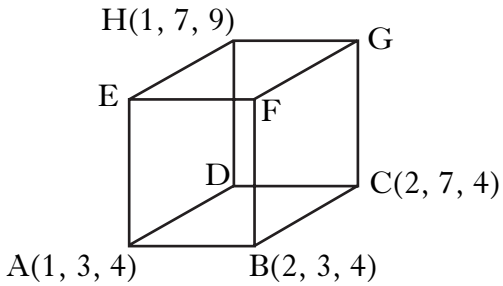
(*b*) Find the equation of the altitude AD. **3**

(*c*) The median from (*a*) and the altitude from (*b*) intersect at P. Find the coordinates of P. **3**

(*d*) The point Q lies on AB and has coordinates (8, 1).

Show that PQ is parallel to BC. **2**

2. The diagram shows a wire framework in the shape of a cuboid with the edges parallel to the axes.



Relative to these axes, A, B, C and H have coordinates (1, 3, 4), (2, 3, 4), (2, 7, 4) and (1, 7, 9) respectively.

(*a*) State the lengths of AB, AD and AE. **1**

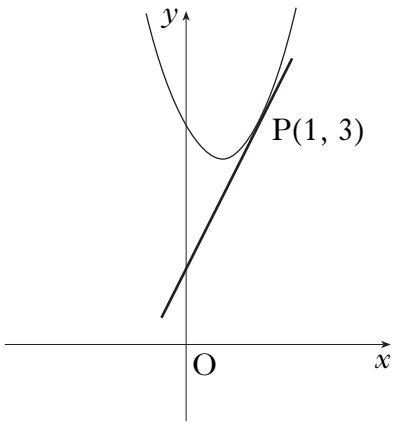
(*b*) Write down the components of $\overrightarrow{HB}$ and $\overrightarrow{HC}$ and hence or otherwise calculate the size of angle BHC. **7**

3. (*a*) Express $5\sin x° - 12\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 < a < 360$. **4**

(*b*) Hence solve the equation $5\sin x° - 12\cos x° = 6{\cdot}5$ in the interval $0 < x < 360$. **3**

Marks

4. The diagram shows a parabola with equation $y = 2x^2 - 2x + 3$.



A tangent to the parabola has been drawn at P(1, 3).

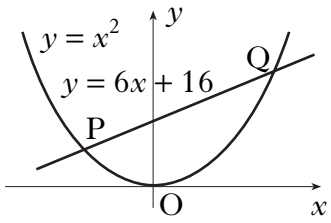
(*a*) Find the equation of this tangent. **4**

A circle has equation $x^2 + y^2 + 8y + 11 = 0$.

(*b*) Show that the line from (*a*) is also a tangent to this circle and state the coordinates of the point of contact Q. **6**

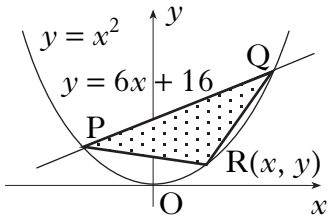
(*c*) Determine the ratio in which the y-axis cuts the line QP. **3**

5. The diagram shows a curve with equation $y = x^2$ and a straight line with equation $y = 6x + 16$ intersecting the curve at P and Q.



(*a*) Calculate the exact value of the area enclosed by the curve and the straight line. **7**

The second diagram shows a third point, R, lying on the curve between P and Q.



(*b*) The area, A, of triangle PQR, is given by $A(x) = -5x^2 + 30x + 80$.

Determine the maximum area of this triangle, and express your answer as a fraction of the area enclosed by the curve and the straight line. **4**

6. Radium decays exponentially and its half-life is 1600 years.

If A_0 represents the amount of radium in a sample to start with and $A(t)$ represents the amount remaining after t years, then $A(t) = A_0e^{-kt}$.

(*a*) Determine the value of k, correct to 4 significant figures. **3**

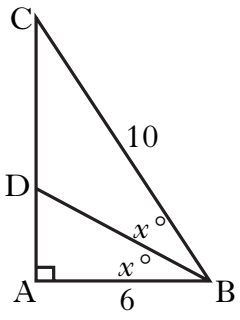
(*b*) Hence find what percentage, to the nearest whole number, of the original amount of radium will be remaining after 3200 years. **2**

Marks

7. Triangle ABC is right-angled at A and BD is the bisector of angle ABC.

AB = 6 units and CB = 10 units.

Determine the exact value of BD, expressing your answer in its simplest form. **5**



[*END OF QUESTION PAPER*]

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[C100/SQP321]

Mathematics
Higher
Paper 1
Specimen Marking Instructions
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (ss), which marks awarded are processing marks (pd) and which marks awarded are interpretation and communication marks (ic).

SCOTTISH QUALIFICATIONS AUTHORITY

SECTION A

1	**B**	$x - x^{-1},\ 1 - x^{-2}, 1 + \frac{1}{x^2}$	**2 marks**
2	**A**	$g(f(x)) = g(2x - 3) = (2x - 3)^2 = 4x^2 - 12x + 9$	**2 marks**
3	**C**	$x^{-\frac{1}{3}},\ \frac{1}{\frac{2}{3}}x^{-\frac{1}{3}+1},\ \frac{3}{2}x^{\frac{2}{3}} + c$	**2 marks**
4	**C**	$d^2 = (2 - -1)^2 + (3 - -4)^2 + (-2 - 0)^2 = 62;\ d = \sqrt{62}$	**2 marks**
5	**A**	$u_1 = 3 \times (-1) - 4 = -7,\ u_2 = 3 \times (-7) - 4 = -25$	**2 marks**
6	**A**		**2 marks**
7	**C**	minimum at $(4, -5)$	**2 marks**
8	**A**	$\sin x° = \frac{2\sqrt{2}}{3},\ \sin 2x° = 2\sin x° \cos x° = 2 \times \frac{2\sqrt{2}}{3} \times \frac{1}{3} = \frac{4\sqrt{2}}{9}$	**2 marks**
9	**B**	$\sin a\cos b - \cos a\sin b = \frac{1}{\sqrt{5}} . \frac{4}{\sqrt{17}} - \frac{2}{\sqrt{5}} . \frac{1}{\sqrt{17}} = \frac{2}{\sqrt{85}}$	**2 marks**
10	**C**	$r^2 = 4^2 + (-3)^2 - (-12) = 37,\ r = \sqrt{37}$	**2 marks**
11	**C**	$\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 10 \\ 6 \end{pmatrix},\ \overrightarrow{QR} = \begin{pmatrix} s-5 \\ 20 \\ 12 \end{pmatrix};\ \ s - 5 = 2 \times 4,\ s = 13$	**2 marks**
12	**B**	$2(x - 3)^2 - 18 + 11,\ c = -7$	**2 marks**
13	**A**	$y = x^3 + \frac{9}{2}x^2 + x + c,\ c = 0$	**2 marks**
14	**D**	$(-3)^2 - 4k = 0;\ \ 4k = 9;\ \ k = \frac{9}{4}$	**2 marks**
15	**A**	$C = (3, 4),\ m_{cp} = \frac{1}{2},\ m_{tgt} = -2$	**2 marks**
16	**C**	$2\sin 2x,\ \ 2\sin\frac{\pi}{3} - 2\sin 0,\ \ 2 \times \frac{\sqrt{3}}{2} - 0 = \sqrt{3}$	**2 marks**

17	**A**	*period* $= 2\pi / 2$, $p = 2$; $q = +1$	**2 marks**
18	**B**	$\boldsymbol{a}.\boldsymbol{c} = 3 \times 5 \times \frac{3}{5} = 9$ *so* (1) T, $\boldsymbol{a}.\boldsymbol{b} = 0$ *so* (2) F	**2 marks**
19	**D**	$\log_3 t - \log_3 5 = 2$, $\log_3\left(\frac{t}{5}\right) = 2$, $\frac{t}{5} = 3^2$, $t = 45$	**2 marks**
20	**C**	$\log_e(3^k) = \log_e(e^4)$, $k\log_e(3) = 4\log_e(e)$, $k = 4 / \log_e(3)$	**2 marks**

[*END OF SECTION A*]

SECTION B

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
21	**(*a*) 2 marks** **(*b*) 4 marks** •[1] ic: interpret "a" and "b" in $au_n + b$ •[2] ic: interpret "a" and "b" in $av_n + b$ •[3] ic: state limit conditions •[4] ss: know how to find limit •[5] pd: process •[6] pd: process and decide	•[1] $u_{n+1} = 0{\cdot}3u_n + 300$ •[2] $v_{n+1} = 0{\cdot}2v_n + 350$ •[3] *limits are valid since* $-1 < 0{\cdot}3 < 1$ *and* $-1 < 0{\cdot}2 < 1$ •[4] A : $L_1 = 0{\cdot}3L_1 + 300$ •[5] A : $L_1 = 428$ *or* 429 •[6] A : $L_1 = 437$ *or* 438 ***and*** product A more effective
22	**(*a*) 7 marks** **(*b*) 4 marks** •[1] ss: know to differentiate •[2] pd: differentiate •[3] ss: set derivative to zero •[4] pd: solve for x •[5] pd: evaluate corresponding y's •[6] ss: know to justify eg use nature table •[7] ic: interpret the stationary points •[8] pd: expand and complete •[9] ic: state x-axis intersections •[10] ic: state y-axis intersections •[11] ic: complete sketch	•[1] $\frac{dy}{dx} = ..$(1 *term correct*) •[2] $\frac{dy}{dx} = 3x^2 - 18x + 24$ •[3] $\frac{dy}{dx} = 0$ •[4] $x = 2$ *or* $x = 4$ •[5] $y = 0$ *or* $y = -4$ •[6] x | … | 2 | … | 4 | … $\frac{dy}{dx}$ | + | 0 | − | 0 | + | ⋰ | … | ⋱ | … | ⋰ •[7] max. *at* (2, 0), min *at* (4, –4) •[8] $y = (x^2 - 4x + 4)(x - 5)$ and complete •[9] x-axis: at (2,0, (5, 0) •[10] y-axis: at (0, –20) •[11] sketch

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
23	**8 marks** •[1] ss: equate functions •[2] pd: rearrange in "standard form" •[3] ss: try a particular value of x •[4] ic: interpret a zero value of equation •[5] ss: start to factorise •[6] pd: obtain quadratic factor •[7] pd: factorise completely •[8] ic: interpret roots	•[1] $x^3 + 5x^2 - 36x + 32 = -x^2 + x + 2$ •[2] $x^3 + 6x^2 - 37x + 30 = 0$ *say* $f(x) = x^3 + 6x^2 - 37x + 30$ •[3] *try evaluating eg* $f(1) = 1^3$ ..... •[4] $f(1) = 1 + 6 - 37 + 30 = 0$ *so* $(x - 1)$ *is a factor* •[5] $(x - 1)(x^2 ....)$ •[6] $(x - 1)(x^2 + 7x - 30)$ •[7] $(x - 1)(x + 10)(x - 3) = 0$ •[8] $x_A = -10$, $x_B = 1$ $x_C = 3$ ***OR*** •[3] $\begin{array}{c\|cccc} 1 & 1 & .. & .. & .. \\ \hline \end{array}$ •[4] $\begin{array}{c\|cccc} 1 & 1 & 6 & -37 & 30 \\ & & 1 & 7 & -30 \\ \hline & 1 & 7 & -30 & 0 \end{array}$ •[5] rem. = 0 *so* $x - 1$ *is factor* •[6] $x^2 + 7x - 30$
24	**5 marks** •[1] ss: know to substitute for $\cos^2 x$ •[2] pd: express in "standard form" •[3] pd: factorise •[4] pd: solve for $\sin p$ •[5] ic: solve for p	•[1] $\sin^2 p - \sin p + 1 = 1 - \sin^2 p$ •[2] $2\sin^2 p - \sin p = 0$ •[3] $\sin p(2\sin p - 1) = 0$ •[4] $\sin p = \frac{1}{2}$, *or* $\sin p = 0$ •[5] $p = \frac{5\pi}{6}$, *no solution in given interval*

[*END OF SECTION B*]

[*END OF SPECIMEN MARKING INSTRUCTIONS*]

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[C100/SQP321]

Mathematics
Higher
Paper 2
Specimen Marking Instructions
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (ss), which marks awarded are processing marks (pd) and which marks awarded are interpretation and communication marks (ic).

SCOTTISH QUALIFICATIONS AUTHORITY

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
1	**(*a*) 3 marks** **(*b*) 3 marks** **(*c*) and (*d*) 5 marks** •[1] ic: interpret "median" •[2] ss: find gradient of median •[3] ic: state equation of median •[4] ss: know to find gradient of "base" •[5] ss: find perpendicular gradient •[6] ic: state equation of altitude •[7] ss: juxtaposition of two equations •[8] pd: solve for one variable •[9] pd: solve for other variable •[10] pd: find gradient of new line •[11] ic: know and use condition for parallel lines	•[1] *mid of* $AB = (6, 1)$ •[2] $m_{median} = \frac{7-1}{4-6} = -3$ •[3] *median* : $y - 7 = -3(x - 4)$ •[4] $m_{BC} = \frac{7-1}{4-10} = -1$ •[5] $m_{alt} = +1$ •[6] *altitude* : $y - 1 = 1(x - 2)$ •[7] $y = -3x + 19$ *and* $y = x - 1$ •[8] $x_p = 5$ •[9] $y_p = 4$ •[10] $m_{PQ} = \frac{4-1}{5-8} = -1$ •[11] $m_{CB} = -1$ *and so* PQ *is parallel to* CB
2	**(*a*) 1 mark** **(*b*) 7 marks** •[1] ic: interpret diagram •[2] ic: interpret diagram for components •[3] ic: interpret diagram for components •[4] pd: evaluate scalar product •[5] pd: evaluate length •[6] pd: evaluate length •[7] ss: know formula for angle •[8] pd: process the angle	•[1] $AB = 1$, $AD = 4$, $AE = 5$ •[2] $\overrightarrow{HB} = \begin{pmatrix} 1 \\ -4 \\ -5 \end{pmatrix}$ •[3] $\overrightarrow{HC} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$ •[4] $\overrightarrow{HB}.\overrightarrow{HC} = 1 + 0 + 25 = 26$ •[5] *magnitude of* HB $= \sqrt{42}$ •[6] *magnitude of* HC $= \sqrt{26}$ •[7] $\cos HBC = (26) / \sqrt{42}\sqrt{26}$ •[8] $H\hat{B}C = 38{\cdot}1\,°$ *or* 0·665 radians

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
3	**(*a*) 4 marks** **(*b*) 3 marks** •[1] ss: know to expand and expand •[2] ic: compare coefficients •[3] pd: process "k" •[4] pd: process "a" •[5] ic: use result •[6] pd: start to solve •[7] pd: complete solving	 •[1] $k\sin x° \cos a° - k\cos x° \sin a°$ •[2] $k\cos a° = 5$, $k\sin a° = 12$ •[3] $k = 13$ •[4] $a = 67{\cdot}4$ •[5] $13\sin(x - 67{\cdot}4)° = 6{\cdot}5$ •[6] $x - 67{\cdot}4 = 30$ *and* $x = 97{\cdot}4$ •[7] $x - 67{\cdot}4 = 150$ *and* $x = 217{\cdot}4$
4	**(*a*) 4 marks** **(*b*) 6 marks** **(*c*) 3 marks** •[1] ss: know to differentiate •[2] pd: differentiate •[3] pd: evaluate gradient of tangent •[4] ic: state equation of tangent •[5] ic: express in standard form •[6] ss: know to substitute •[7] pd: process into standard form •[8] ss: know how to prove tangency •[9] ic: complete proof •[10] pd: evaluate point of contact •[11] ss: know to find coord of intersection •[12] ss: choose approach (eg vectors) •[13] ic: interpret ratio (from eg vectors)	 •[1] $\frac{dy}{dx} = \ldots$ •[2] $\frac{dy}{dx} = 4x - 2$ •[3] $m_{x=1} = 2$ •[4] $y - 3 = 2(x - 1)$ •[5] $y = 2x + 1$ •[6] $x^2 + (2x + 1)^2 + 8(2x + 1) + 11 = 0$ •[7] $5x^2 + 20x + 20 = 0$ •[8] $5(x + 2)^2 = 0$ •[9] *equal roots so line is tangent* •[10] Q = (–2, –3) •[11] *line cuts y – axis at* (0, 1)(= *T, say*) •[12] $\overrightarrow{QT} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\overrightarrow{TP} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ •[13] $\overrightarrow{QT} = 2\overrightarrow{TP}$ *and* QT : TP = 2 : 1

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
5	**(*a*) 7 marks** **(*b*) 4 marks** •[1] ss: know to equate •[2] pd: solve for x •[3] ss: know to integrate (upper–lower) •[4] ic: interpret limits •[5] pd: integrate •[6] ic: substitute limits •[7] pd: process results •[8] ss: set derivative to zero •[9] pd: differentiate and solve •[10] ic: know to justify (eg know of maximum parabola) •[11] ic: interpret maximum area and connection	•[1] $x^2 = 6x + 16$ •[2] $(x-8)(x+2) = 0$ *and* $x = 8,\ x = -2$ •[3] $A = \int (6x + 16 - x^2)dx$ •[4] $A = \int_{-2}^{8} \ldots\, dx$ •[5] $3x^2 + 16x - \frac{1}{3}x^3$ •[6] $\left[192 + 128 - \frac{512}{3}\right] - \left[12 - 32 + \frac{8}{3}\right]$ •[7] $\frac{500}{3}$ •[8] $\frac{dA}{dx} = 0$ •[9] $x = 3$ •[10] *nature table or* "-5" $\Rightarrow$ maximum parabola •[11] triangle $\text{area}_{\max} = 125 = \frac{3}{4}$ of enclosed area
6	**5 marks** •[1] ic: interpret half-life •[2] ss: take logarithms •[3] pd: complete evaluation of "k" •[4] ic: substitute new data •[5] pd: process	•[1] $\frac{1}{2}A_0 = A_0 e^{-1600k}$ •[2] $-1600k = \ln\frac{1}{2}$ •[3] $k = 0{\cdot}0004332$ •[4] $A(3200) = A_0 e^{-3200k}$ •[5] $A(3200) = A_0 \times 0{\cdot}2500$ ***and*** 25%
7	**5 marks** •[1] ic: state value of trig function •[2] ss: substitute expression for $\cos 2x$ •[3] pd: process •[4] ss: know to review info in diagram •[5] pd: equate expressions and solve	•[1] $\cos(2x) = \frac{6}{10}$ •[2] $2\cos^2(x) - 1 = \frac{6}{10}$ •[3] $\cos(x) = \sqrt{\frac{4}{5}}$ •[4] $\cos(x) = \frac{6}{DB}$ •[5] $\text{DB} = 6\sqrt{\frac{5}{4}} = 3\sqrt{5}$

[*END OF SPECIMEN MARKING INSTRUCTIONS*]

Specimen Question Paper : Summary Analysis

Specimen Question Paper 1 Section A

qu	ans	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1	B	2	C3	CN	8005	1	1		2				2		
2	A	2	A4	CN	8018		1	1	2				2		
3	C	2	C13	CN	8020	1	1		2					2	
4	D	2	G1	CN	8035	1	1		2				2		
5	A	2	A11	CN	8010		1	1	2				2		
6	A	2	A3	CN	8009			2	2				2		
7	C	2	A6	NC	8008			2	2				2		
8	A	2	T8	CN	8006	1	1		2					2	
9	B	2	T9	CN	8016	1	1		2					2	
10	C	2	G9	CN	8012		1	1	2					2	
11	C	2	G23+,G2	CN	8017		1	1	2						2
12	B	2	A5	CN	8002		2		2				2		
13	A	2	A28,A31	CN	8004	1	1		2						2
14	D	2	A18	CN	8001	1	1		2					2	
15	A	2	G11	CN	8007	1		1	2					2	
16	C	2	C23,T3	NC	8003		2		2						2
17	A	2	T1	NC	8019			2	2				2		
18	B	2	G26,G27	CN	8041	1		1		2					2
19	D	2	A28	CN	8015	1	1			2					2
20	C	2	A34	CN	8014	1	1			2					2
					totals	11	17	12	34	6	0	**40**	16	12	12

Specimen Question Paper 1 Section B

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
21	a	2	A10	CN	8022			2	2				2		
	b	4	A14,A12,A13			1	2	1	4				4		
22	a	7	C8,9	NC	8025	3	3	1	7				7		
	b	4	C10				1	3	1	3			4		
23		8	A25,A21,A22	NC	8037	3	3	2	8					8	
24		5	T7,T3	NC	8024	1	3	1		5				5	
					totals	8	12	10	22	8	0	**30**	17	13	0

Specimen Question Paper 2

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A		U1	U2	U3
1	a	3	G7,G6,G3	CN	8026	1		2	3				3		
	b	3	G7,G5,G3	CN		2		1	3				3		
	c	5	G23+, G2	CN		1	3	1	5				5		
2	a	1	–	CN	8027			1	1						1
	b	7	G17,G26,G28	CR		1	4	2	7						7
3	a	4	T13,T12	CR	8029	1	2	1	4						4
	b	3	T16	CR			2	1	1	2					3
4	a	4	C5,C4	CN	8028	1	2	1	4				4		
	b	6	G12,G13,A17	CN		2	2	2	2	4				6	
	c	3	G24	CN		2		1			3			3	
5	a	7	C17,C15,A23	CN	8034	2	3	2		7				7	
	b	4	C11	CN		1	1	2		3	1			3	1
6		5	A30,A34	CR	8031	1	2	2			5				5
7		5	T11,T7	CN	8036	2	2	1			5			5	
					totals	17	23	20	30	16	14	**60**	15	24	21
					TOTAL	**36**	**52**	**42**	**86**	**30**	**14**	**130**	**48**	**49**	**33**
					target	33+	33+	33+	85±1	2:1			33+	33+	33+