[C100/SQP328]

Mathematics
Higher
Paper 1

Specimen Question Paper
Example 2 based on 2004 Examination Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

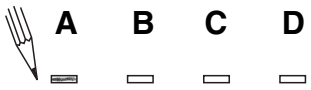
A 8

B 1

C 0

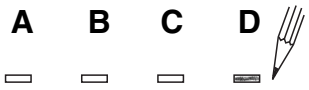
D −4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

1. The line through P(7, p) and Q(4, –5) has a gradient of 3.
What is the value of p?

A –14

B 4

C 6

D 8

2. A sequence is defined by the recurrence relation $u_{n+1} = u_n + 5,\ u_0 = -3$.
What is the value of u_2?

A 3
B 5
C 7
D 9

3. What is the gradient of the line perpendicular to the line with equation $3y = -2x + 1$?

A –3

B 1

C $\frac{3}{2}$

D 5

4. $f(x) = x^3 - x^2 - 5x - 3$.
What is the remainder when $f(x)$ is divided by $(x + 3)$?

A –24
B –3
C 36
D 48

5. If $x^2 - 16x + 27$ is written in the form $(x + p)^2 + q$, find the value of q.

A -37

B 11

C 27

D 43

6. What is the derivative of $(8 - 2x^2)^{\frac{2}{3}}$?

A $-\frac{8}{3}x(8 - 2x^2)^{-\frac{1}{3}}$

B $(8 - 4x)^{\frac{2}{3}}$

C $\frac{2}{3}(8 - 4x)^{-\frac{1}{3}}$

D $\frac{3}{5}(8 - 2x^2)^{\frac{5}{3}}$

7. On dividing $f(x)$ by $(x - 1)$, the remainder is zero and the quotient is $x^2 - 4x - 5$.
Find $f(x)$ in its fully factorised form.

A $(x - 1)(x - 1)(x + 5)$

B $(x + 1)(x - 5)$

C $(x - 1)(x - 1)$

D $(x - 1)(x + 1)(x - 5)$

8. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n + 3$.
What is the limit of this sequence as $n \to \infty$?

A $\frac{1}{5}$

B $\frac{15}{7}$

C 5

D $\frac{15}{2}$

9. Find all the values of x in the interval $0 < x < 2\pi$ for which $\tan x = -\sqrt{3}$.

A $\frac{5\pi}{6}, \frac{11\pi}{6}$

B $\frac{2\pi}{3}, \frac{4\pi}{3}$

C $\frac{2\pi}{3}, \frac{5\pi}{3}$

D $\frac{5\pi}{3}, \frac{7\pi}{3}$

10. P = (–3, 4, 7), Q = (–1, 8, 3) and R = (0, 10, 1).

Find the ratio in which Q divides PR.

A 2 : 1

B 3 : –1

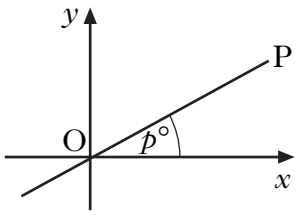
C 1 : 2

D 3 : 1

11. The diagram shows the line OP with equation $2y = x$.

The angle between OP and the positive direction of the x-axis is $p°$.

Find an expression for angle p.



A $\tan^{-1}\frac{1}{2}$

B $\tan^{-1}1$

C $\tan^{-1}2$

D $-\tan^{-1}\frac{1}{2}$

12. Which one of the following is true for the function g where $g'(x) = x^2 + 2x + 1$?

A g is never increasing.

B g is decreasing then increasing.

C g is increasing then decreasing.

D g is never decreasing.

13. Simplify $\log_2(x + 1) - 2\log_2 3$.

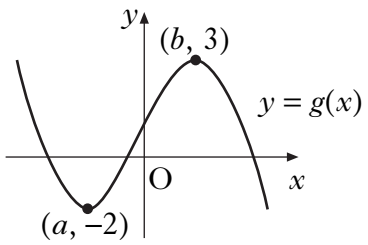
A $\log_2\left(\frac{x+1}{9}\right)$

B $\log_2(x - 8)$

C $\log_2(x - 2)$

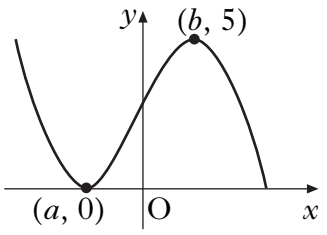
D $\log_2 6(x + 1)$

14. The diagram shows the graph of $y = g(x)$.

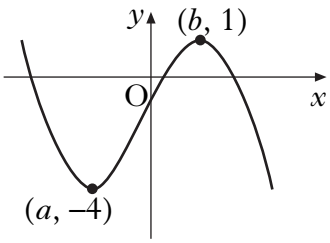


Which diagram below shows the graph of $y = 3 - g(x)$?

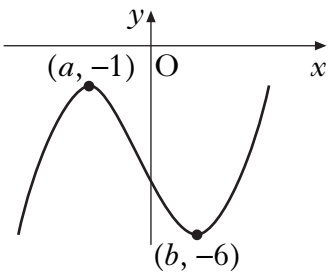
A



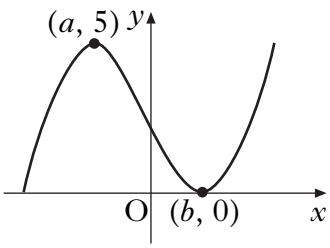
B



C



D



15. Points P and Q have coordinates (1, 3, –1) and (2, 5, 1) and T is the midpoint of PQ. What is the position vector of T?

A $\begin{pmatrix} -\frac{3}{2} \\ -4 \\ 0 \end{pmatrix}$

B $\begin{pmatrix} \frac{3}{2} \\ 4 \\ 0 \end{pmatrix}$

C $\begin{pmatrix} -\frac{1}{2} \\ -1 \\ -1 \end{pmatrix}$

D $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$

16. A = (–3, 4, 7) and B = (–1, 8, 3).
If $\overrightarrow{AD} = 4\overrightarrow{AB}$, what are the coordinates of D?

A (–9, –8, –13)

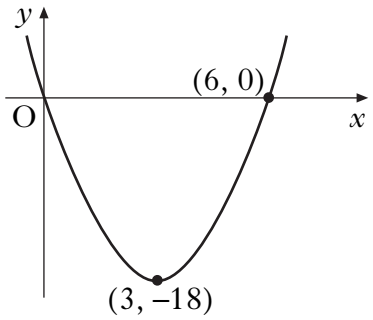
B (5, –4, 1

C (–6, 8, 14)

D (5, 20, –9)

17. The equation of the parabola shown is of the form $y = kx(x - 6)$.

What is the value of k?



A 0

B $\frac{1}{144}$

C 2

D 6

18. Given that $y = 3\cos 5x$, find $\frac{dy}{dx}$.

A $15\cos 5x$

B $-15\sin 5x$

C $-15\cos x$

D $3\cos 5$

19. Find $\int (4x+1)^{\frac{1}{2}}\, dx$.

A $\frac{1}{6}(4x+1)^{\frac{3}{2}} + c$

B $\frac{1}{4}(4x+1) + c$

C $\frac{1}{4}(4x+1)^{\frac{3}{2}} + c$

D $2(4x+1)^{-\frac{3}{2}} + c$

20. Given that $\int (3x+1)^{-\frac{1}{2}}\,dx = \frac{2}{3}(3x+1)^{\frac{1}{2}} + c$, find $\int_0^1 (3x+1)^{-\frac{1}{2}}\,dx$.

A $\frac{2}{3}$

B $\frac{4}{3}$

C 2

D $\sqrt{2}$

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. (*a*) Find the stationary points on the curve with equation $y = x^3 + 3x^2 - 9x + 5$ and justify their nature. **7**

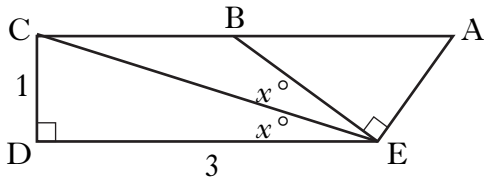
(*b*) The curve passes through the point (–5, 0). Sketch the curve. **2**

22. Solve the equation $\log_x 8 + \log_x 4 = 5$. **4**

23. Solve the equation $\sin 2x - \cos x = 0$ for $0 \leq x \leq 2\pi$. **5**

24. In the diagram,

angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.



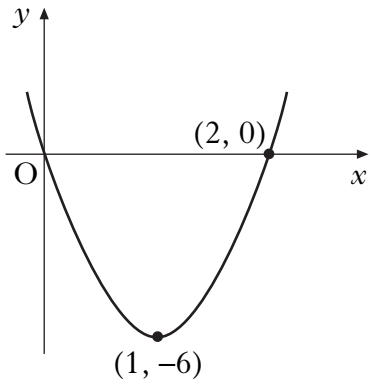
By writing angle DEA in terms of $x°$, find the exact value of cos (DÊA). **7**

25. The diagram shows a parabola with equation

$$y = 6x(x - 2).$$

This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$. **5**



[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

[C100/SQP328]

Mathematics
Higher
Paper 2

Specimen Question Paper
Example 2 based on 2004 Examination Paper
(for examinations from Diet 2008 onwards)

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FORMULAE LIST

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Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
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Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
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$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Given that $\overrightarrow{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$, find the size of angle PQR. **5**

2. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

3. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 12. **5**

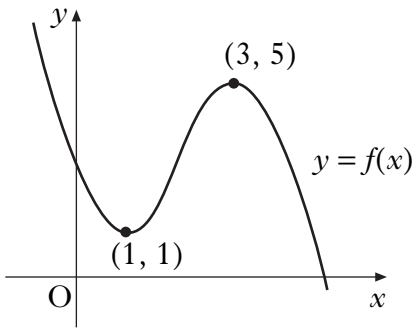
(b) Hence find the equation of the tangent at P. **2**

4. (a) Express $3\cos x° + 5\sin x°$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4**

(b) Hence solve the equation $3\cos x° + 5\sin x° = 4$ for $0 \leq x \leq 90$. **3**

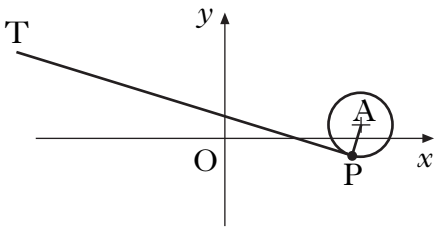
5. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5).

Sketch the graph of $y = f'(x)$. **3**



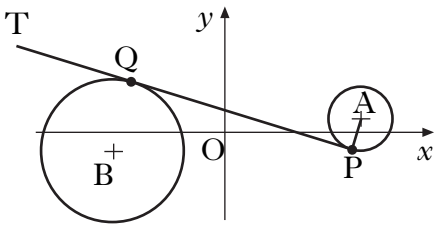
Marks

6. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point P(5, –1).



(*a*) Show that the equation of this tangent is $x + 2y = 3$. **4**

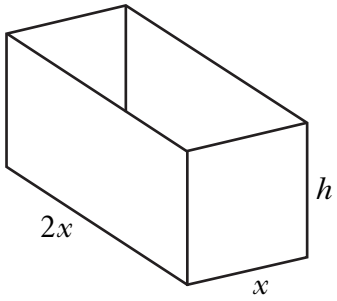
The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.



(*b*) Show that PT is also a tangent to this circle. **5**

(*c*) Q is the point of contact. Find the length of PQ. **2**

7. An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.



(*a*) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$. **3**

(*b*) Find the exact value of x for which this volume is a maximum. **5**

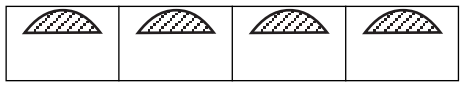
8. The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0 e^{-0{\cdot}002t}$, where A_0 is the amount present initially.

(*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3**

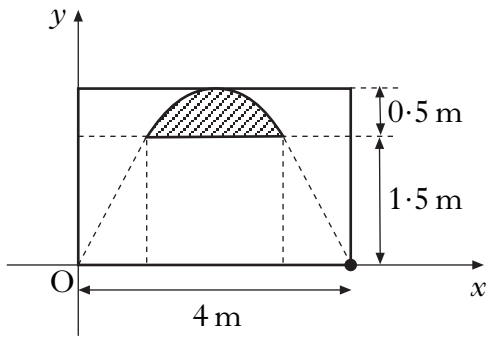
(*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**

Marks

9. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.



The second diagram shows one such window. The shaded part represents the glass.



The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window. **8**

[*END OF QUESTION PAPER*]

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[C100/SQP328]

Mathematics
Higher
Paper 1
Specimen Marking Instructions
Example 2 based on 2004 Examination Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (ss), which marks awarded are processing marks (pd) and which marks awarded are interpretation and communication marks (ic).

SECTION A

1	**B**	$m = \frac{p-(-5)}{7-4}$ $3 = \frac{p+5}{3} \Rightarrow p = 4$	**2 marks**
2	**C**	$u_1 = u_0 + 5 = -3 + 5 = 2$ $u_2 = u_1 + 5 = 2 + 5 = 7$	**2 marks**
3	**C**	$y = -\frac{2}{3}x + \frac{1}{3},\ m = -\frac{2}{3}$ $m = \frac{3}{2}$	**2 marks**
4	**A**	$f(-3) = (-3)^3 - (-3)^2 - 5(-3) - 3$ $f(-3) = -24$	**2 marks**
5	**A**	$x^2 - 16x + 64 - 37$ $(x-8)^2 - 37$ $q = -37$	**2 marks**
6	**A**	$\frac{2}{3}(8-2x^2)^{-\frac{1}{3}} \times (-4x)$ $\frac{-8x}{3(8-2x^2)^{\frac{1}{3}}}$	**2 marks**
7	**D**	$f(x) = (x-1)(x^2-4x-5)$ $f(x) = (x-1)(x-5)(x+1)$	**2 marks**
8	**C**	$L = "\frac{b}{1-a}"$ $L = \frac{3}{1-0\cdot 4} = 5$	**2 marks**
9	**C**	$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$	**2 marks**

10	**A**	$\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{PQ} = 2\overrightarrow{QR}$ PQ:QR = 2:1	**2 marks**
11	**A**	$y = \frac{1}{2}x$, *gradient* $= \frac{1}{2}$ $p = \tan^{-1}\left(\frac{1}{2}\right)$	**2 marks**
12	**D**	$g'(x) = (x + 1)^2$ $g'(x) \geq 0$ for all x so g is never decreasing	**2 marks**
13	**A**	$\log_2(x + 1) - \log_2 9$ $\log_2\left(\frac{x+1}{9}\right)$	**2 marks**
14	**D**	reflect original in the axis; then translate this image upwards by 3 units	**2 marks**
15	**B**	$\overrightarrow{OT} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$ $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ so $\overrightarrow{OT} = \begin{pmatrix} \frac{3}{2} \\ 4 \\ 0 \end{pmatrix}$	**2 marks**
16	**D**	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$ A = (–3, 4, 7) and so D = (5, 20, –9)	**2 marks**
17	**C**	substitute (3, –18) $-18 = k \times 3 \times (3 - 6)$ $\Rightarrow k = 2$	**2 marks**

18	**B**	$\frac{dy}{dx} = 3 \times -\sin(5x) \times 5$ $\frac{dy}{dx} = -15\sin(5x)$	 **2 marks**
19	**A**	$\frac{1}{\frac{3}{2}} \times (4x+1)^{\frac{3}{2}}$ *divided by* 4 $\frac{1}{6}(4x+1)^{\frac{3}{2}} + c$	 **2 marks**
20	**A**	$\frac{2}{3}(3+1)^{\frac{1}{2}} - \frac{2}{3}(0+1)^{\frac{1}{2}}$ $\frac{2}{3} \times \sqrt{4} - \frac{2}{3} \times \sqrt{1} = \frac{2}{3}$	 **2 marks**

[*END OF SECTION A*]

SECTION B

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
21	**(*a*) 7 marks** **(*b*) 2 marks** •[1] ss: know to differentiate •[2] pd: differentiate •[3] ss: know to set derivative to zero •[4] pd: process •[5] pd: process •[6] ic: justify using eg nature table •[7] ic: interpret nature table •[8] ic: sketch •[9] ic: sketch	•[1] $\frac{dy}{dx} =$ •[2] $3x^2 + 6x - 9$ •[3] $3x^2 + 6x - 9 = 0$ •[4] $x = -3,\ x = 1$ •[5] $y = 32,\ y = 0$ •[6] "*nature table*" •[7] max at $x = -3$, min at $x = 1$ 7 *marks* •[8] diagram with max/min correct •[9] diagram with (0, 5) and (–5, 0) correct 2 *marks*
22	**4 marks** •[1] ss: use the log laws •[2] ss: transfer from log to exponential •[3] pd: start to solve equation •[4] pd: complete solving	•[1] $\log_x 32 = 5$ •[2] $32 = x^5$ •[3] $x = \sqrt[5]{32}$ •[4] $x = 2$ 4 *marks*
23	**5 marks** •[1] ss: know to use double angle formula •[2] pd: factorise •[3] pd: solve •[4] ic: interpret solutions •[5] ic: interpret solutions	•[1] $2\sin x \cos x - \cos x = 0$ •[2] $\cos x(2\sin x - 1) = 0$ •[3] $\cos x = 0 \qquad \sin x = \frac{1}{2}$ •[4] $x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$ •[5] $x = \frac{3\pi}{2} \qquad x = \frac{5\pi}{6}$ 5 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
24	**7 marks** •[1] ic: interpret diagram •[2] pd: expand trig expression •[3] pd: simplify •[4] ss: use appropriate formula •[5] pd: process •[6] ic: interpret •[7] pd: simplify	•[1] $D\hat{E}A = (2x° + 90°)$ •[2] $\cos(2x°)\cos(90°) - \sin(2x°)\sin(90°)$ •[3] $-\sin(2x°)$ •[4] $-2\sin(x°)\cos(x°)$ •[5] $CE = \sqrt{1^2 + 3^2} = \sqrt{10}$ *stated/implied by* •[6] •[6] $\sin(x°) = \left(\frac{1}{\sqrt{10}}\right)$ *and* $\cos(x°) = \frac{3}{\sqrt{10}}$ •[7] $\cos D\hat{E}A = -2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) = -\frac{6}{10}$ *7 marks*
		Note 1 Although unusual, it would be perfectly acceptable for a candidate to go from •[1] to •[3] without expanding (via knowledge of transformations). In this case •[2] would be awarded by default.

1 common wrong solution	2 another common wrong solution
•[1]✓ $D\hat{E}A = (2x° + 90°)$ •[2]✓ $\cos(2x°)\cos(90°) - \sin(2x°)\sin(90°)$ $\cos(2x°) \times 1 - \sin(2x°) \times 0$ •[3]✘ $\cos(2x°)$ •[4]✓ *eg* $2\cos^2 x - 1$ •[5]✓ $CE = \sqrt{1^2 + 3^2} = \sqrt{10}$ *stated/implied by* •[6] •[6]✓ $\cos(x°) = \frac{3}{\sqrt{10}}$ •[7]✓ $\cos D\hat{E}A = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) - 1 = \frac{8}{10}$ **6 marks awarded**	•[1]✓ $D\hat{E}A = (2x° + 90°)$ $\cos(2x° + 90°)$ •[2]✘ $\cos(2x°) + \cos(90°)$ •[3]✘ $\cos(2x°)$ [*working eased*] •[4]✓ *eg* $2\cos^2 x - 1$ •[5]✓ $CE = \sqrt{1^2 + 3^2} = \sqrt{10}$ *stated/implied by* •[6] •[6]✓ $\cos(x°) = \frac{3}{\sqrt{10}}$ •[7]✓ $\cos D\hat{E}A = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) - 1 = \frac{8}{10}$ **5 marks awarded**

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
25	**5 marks** •[1] ss: know to integrate •[2] pd: express in integrable form •[3] pd: interpret •[4] ss: introduce constant and substitute •[5] pd: process	•[1] $f(x) = \int (6x(x-2))\,dx$ •[2] $\int (6x^2 - 12x)\,dx$ •[3] $2x^3 - 6x^2$ •[4] $4 = 2 \times 1^3 - 6 \times 1^2 + c$ •[5] $c = 8$ 5 *marks*
		Note 1 $\int_0^2 6x(x-2)dx = \left[2x^3 - 6x^2\right]_0^2 = -8$ may be awarded •[1], •[2] and •[3].

[*END OF SECTION B*]

[*END OF SPECIMEN MARKING INSTRUCTIONS*]

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[C100/SQP328]

Mathematics
Higher
Paper 2
Specimen Marking Instructions
Example 2 based on 2004 Examination Paper
(for examinations from Diet 2008 onwards)

NATIONAL QUALIFICATIONS

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (ss), which marks awarded are processing marks (pd) and which marks awarded are interpretation and communication marks (ic).

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
1	**5 marks** •[1] ss: know to use eg scalar product •[2] pd: process scalar product •[3] pd: process length •[4] pd: process length •[5] pd: process angle	•[1] $\cos \mathrm{PQR} = \dfrac{\overrightarrow{\mathrm{QP}}.\overrightarrow{\mathrm{QR}}}{\lvert\overrightarrow{\mathrm{QP}}\rvert \times \lvert\overrightarrow{\mathrm{QR}}\rvert}$ stated or implied by •[5] •[2] $\overrightarrow{\mathrm{QP}}.\overrightarrow{\mathrm{QR}} = 6$ •[3] $\lvert\overrightarrow{\mathrm{QP}}\rvert = \sqrt{14}$ •[4] $\lvert\overrightarrow{\mathrm{QR}}\rvert = \sqrt{27}$ •[5] $\mathrm{P\hat{Q}R} = 72 \cdot 0°$ 5 *marks*

Alternative Method

•[1] $\cos \mathrm{P\hat{Q}R} = \dfrac{p^2 + r^2 - q^2}{2pr}$ stated or implied by •[5]

•[2] $q = \sqrt{29}$

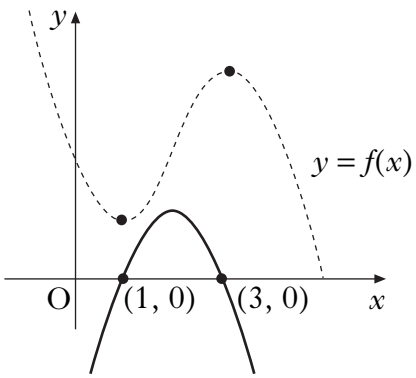
•[3] $r = \sqrt{14}$

•[4] $p = \sqrt{27}$

•[5] $\mathrm{P\hat{Q}R} = 72 \cdot 0°$

5 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
2	**4 marks** •[1] ss: know/use discriminant •[2] ic: identify discriminant •[3] pd: simplify •[4] ic: complete proof	•[1] *know to show* $b^2 - 4ac \geq 0$ •[2] $p^2 - 4 \times 2 \times (-3)$ •[3] $p^2 + 24$ •[4] p^2 *is positive* *so* $\Delta \geq 0$ *and roots real* 4 *marks*
		Alternative Method •[1] $x = \dfrac{-p \pm \sqrt{(-p)^2 - 4\times 2\times (-3)}}{4}$ •[2] $x = \dfrac{-p \pm \sqrt{p^2 + 24}}{4}$ •[3] *we need* $p^2 + 24 \geq 0$ •[4] p^2 *is positive and so roots real* 4 *marks*
3	**(*a*) 5 marks** **(*b*) 2 marks** •[1] ss: know to differentiate •[2] pd: differentiate •[3] ss: set derivative = gradient •[4] pd: start to solve •[5] pd: process •[6] pd: process •[7] ic: state equation of tangent	•[1] $\dfrac{dy}{dx} =$ *stated or implied by* •[2] •[2] $12x - 3x^2$ •[3] $12x - 3x^2 = 12$ •[4] $3(x-2)^2 = 0$ •[5] $x = 2$ 5 *marks* •[6] $y = 16$ •[7] $y - 16 = 12(x-2)$ 2 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
4	**(*a*) 4 marks** **(*b*) 3 marks** •[1] ss: expand •[2] ic: equate coefficients •[3] pd: solve for k •[4] pd: solve for a •[5] ss: use transformed function •[6] pd: solve trig equation for "$x - a$" •[7] pd: solve for x	•[1] $k\cos x\cos a + k\sin x\sin a$ *STATED EXPLICITLY* •[2] $k\cos a = 3,\ k\sin a = 5$ *STATED EXPLICITLY* •[3] $k = \sqrt{34}$ •[4] $a = 59$ 4 *marks* •[5] $\sqrt{34}\cos(x - 59)° = 4$ •[6] $x - 59$ = any one of $-46{\cdot}7,\ 47{\cdot}7,\ 313{\cdot}3$ •[7] $x = 12{\cdot}3$ 3 *marks*
5	**3 marks** •[1] ic: interpret stationary points •[2] ic: interpret between roots •[3] ic: know $f'(cubic) = parabola$  	a sketch with the following details •[1] *only two intercepts on the x-axis at 1 and 3* •[2] *function is +ve between the roots and −ve outwith* •[3] *a parabola (symmetrical about midpoint of x-intercepts), stated or implied by the accuracy of the diagram* 3 *marks* **Note** 1 The evidence for •[1] may be on a diagram or in a table or in words 2 For •[3], with the intercepts unknown, they must lie on the positive branch of the x-axis 3 For a parabola passing through (1, 1) and (3, 5) award **ONLY 1 MARK**

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
6	**(*a*) 4 marks** **(*b*) 5 marks** **(*c*) 2 marks** •[1] ic: interpret circle equation •[2] ic: find gradient •[3] ss: know/find perpendicular gradient •[4] pd: complete proof •[5] pd: start solving process •[6] ss: know/substitute •[7] pd: arrange in standard form •[8] ss: know how to justify tangency •[9] ic: complete proof •[10] ic: interpret solution from (*b*) •[11] pd: process distance formula	•[1] $A(6, 1)$ •[2] $m_{AP} = 2$ *STATED EXPLICITLY* •[3] $m_{PT} = -\frac{1}{2}$ •[4] $y + 1 = -\frac{1}{2}(x - 5)$ *and complete* 4 *marks* •[5] $x = 3 - 2y$ •[6] $(3 - 2y)^2 + y^2 + 10(3 - 2y) + 2y + 6 = 0$ •[7] $5y^2 - 30y + 45 = 0$ •[8] *solve and get double root* $\Rightarrow$ *tangent* •[9] $5(y - 3)^2 = 0$ 5 *marks* •[10] $Q = (-3, 3)$ •[11] $PQ = \sqrt{80}$ 2 *marks*
		Alternative method for •[1] to •[4] •[1] $(3 - 2y)^2 + y^2 - 12(3 - 2y) - 2y + 32 = 0$ •[2] $5(y + 1)^2 = 0$ •[3] *double root* $\Rightarrow$ *tangent* •[4] $x = 3 - 2y = 3 - 2 \times (-1) = 5$ 4 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
7	**(*a*) 3 marks** **(*b*) 5 marks** •[1] ss: use area facts •[2] ss: use volume facts •[3] ic: complete proof •[4] pd: arrange in standard form •[5] pd: differentiate •[6] ss: set derivative to zero •[7] pd: process •[8] ic: justification eg a nature table	•[1] $A = 2x^2 + 2xh + 4xh = 12$ •[2] $V = 2x \times x \times h$ •[3] $V = 2x \times \frac{12-2x^2}{6} = \text{and complete}$ 3 *marks* •[4] $V = 4x - \frac{2}{3}x^3$ •[5] $\frac{dV}{dx} = 4 - 2x^2$ •[6] $\frac{dV}{dx} = 0$ *STATED EXPLICITLY* •[7] $x = \sqrt{2}$ •[8] x | $< \sqrt{2}$ | $\sqrt{2}$ | $> \sqrt{2}$ $\frac{dV}{dx}$ | $+ve$ | 0 | $-ve$ tgt | / | − | \ max 5 *marks*

Alternative method for •[1], •[2] and •[3]

•[1] $2x^2 + 2xh + 4xh = 12$
•[2] $h = \frac{12-2x^2}{6}$
•[3] $V = 2x \times x \times \frac{12-2x^2}{6x} = \text{and complete}$
3 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
8	**(*a*) 3 marks** **(*b*) 4 marks** •[1] ss: substitute •[2] pd: change the subject •[3] pd: process exponential power •[4] ic: interpret half life •[5] pd: process •[6] ss: switch to logarithmic form •[7] pd: solve logarithmic equation	•[1] $600 = A_0 e^{-0\cdot002\times1000}$ •[2] $A_0 = \dfrac{600}{e^{-0\cdot002\times1000}}$ •[3] 4433 3 *marks* •[4] $\frac{1}{2}A_0 = A_0 e^{-0\cdot002t}$ •[5] $0\cdot5 = e^{-0\cdot002t}$ •[6] $-0\cdot002t = \ln 0\cdot5$ •[7] $t = 347$ years 4 *marks*
		Alternative method for (*a*) •[1] $600 = A_0 e^{-0\cdot002\times1000}$ •[2] $\ln A_0 = \ln 600 - \ln e^{-0\cdot002\times1000}$ •[3] $A_0 = 4433$ 3 *marks*

Qu	The Primary Method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.	Primary Method: Give 1 mark for each •
9	**8 marks** •[1] ss: find intersections •[2] pd: process quadratic to solution •[3] ss: decide on appropriate areas •[4] ss: know to integrate •[5] ic: state limits •[6] pd: integrate •[7] pd: evaluate using limits •[8] pd: evaluate area	•[1] $2x - \frac{1}{2}x^2 = 1 \cdot 5$ •[2] $x = 1,\ x = 3$ •[3] "*split area up*" *stated or implied by* •[4] •[4] $\int \left(2x - \frac{1}{2}x^2 - \frac{3}{2}\right)dx$ •[5] $\int_1^3 \ldots dx$ •[6] $\left[x^2 - \frac{1}{6}x^3 - \frac{3}{2}x\right]_1^3$ •[7] $\left(3^2 - \frac{1}{6}.3^3 - \frac{3}{2}.3\right) - \left(1^2 - \frac{1}{6}.1^3 - \frac{3}{2}.1\right)$ •[8] $\frac{2}{3}$ 8 *marks*

[*END OF SPECIMEN MARKING INSTRUCTIONS*]