Mathematics	Time allowed	NATIONAL
Higher	1 hour 30 minutes	QUALIFICATIONS
Paper 1		
Practice Paper A		

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.

2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1   Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2   For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3   Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4   If any of this information is wrong, tell the invigilator immediately.

5   If this information is correct, print your name and seat number in the boxes provided.

6   The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7   There is **only one correct** answer to each question.

8   Rough working should **not** be done on the answer sheet.

9   At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

    A   8

    B   1

    C   0

    D −4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).

Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product : $\quad \boldsymbol{a} . \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$\text{or} \quad \boldsymbol{a} . \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3, \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Trigonometric formulae:
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

1.   A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 1$, $u_0 = 3$.

What is the value of u_2?

A   1

B   5

C   11

D   15

2.   The line with equation $kx - 2y + 9 = 0$ is parallel to the line with gradient 7.

What is the value of k?

A   −7

B $-\dfrac{1}{7}$

C   7

D   14

3.   A circle has equation $x^2 + y^2 - 8x + 2y - 1 = 0$.

What is the radius of this circle?

A $\sqrt{6}$ units

B $\sqrt{7}$ units

C $\sqrt{18}$ units

D $\sqrt{69}$ units

4. What is the derivative of $\dfrac{x^3 - 2}{3x}$ with respect to x?

A $\dfrac{2}{3}x + \dfrac{2}{3}x^{-2}$

B $6x + 6x^{-2}$

C $2x^2$

D $\dfrac{2}{3}x^3 + \dfrac{2}{3}$

5. Find $\displaystyle\int \dfrac{1}{2x^4}\,dx.$

A $-\dfrac{1}{8x^3} + c$

B $-\dfrac{1}{6x^3} + c$

C $\dfrac{1}{8x^3} + c$

D $\dfrac{5}{2x^5} + c$

6. If $x^2 - 12x + 37$ is written in the form $(x - p)^2 + q$, find the value of q.

A   1

B   25

C   31

D   37

7. A sequence is generated by the recurrence relation $u_{n+1} = 0 \cdot 8u_n + 16$.

What is the limit of this sequence as $n \to \infty$?

A   2

B $13\dfrac{1}{4}$

C   16

D   80

8. A circle with centre $(-1, 5)$ passes through the point $(2, 7)$.

   What is the equation of the circle?

   A $(x-1)^2 + (y+5)^2 = 145$

   B $(x-1)^2 + (y+5)^2 = 13$

   C $(x+1)^2 + (y-5)^2 = 145$

   D $(x+1)^2 + (y-5)^2 = 13$

9. The vectors p and q with components $p = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ and $q = \begin{pmatrix} k \\ -3 \\ -2 \end{pmatrix}$ are perpendicular.

   What is the value of k?

   A -2

   B -1

   C 0

   D 1

10. Here are two statements about the equation $2x^2 - 8x + 3 = 0$

    (1) The roots are real;

    (2) The roots are irrational.

    Which of the following is true?

    A Neither statement is correct.

    B Only statement (1) is correct.

    C Only statement (2) is correct.

    D Both statements are correct.

11. What is the value of $\cos\dfrac{5\pi}{3} - \tan\dfrac{7\pi}{4}$?

    A -1

    B $-\dfrac{1}{2}$
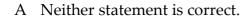
    C $\dfrac{\sqrt{3}}{2}$

    D $\dfrac{3}{2}$

12. Given that $\log_2 \frac{1}{8} = p$, find the value of p.

    A -3

    B -2

    C $\frac{1}{64}$

    D   3

13. Find $\int (3x-11)^5 \, dx$

    A $\frac{1}{15}(3x-11)^4 + c$

    B $\frac{1}{2}(3x-11)^4 + c$

    C $\frac{1}{18}(3x-11)^6 + c$

    D $15(3x-11)^6 + c$

14. K and L are the points with coordinates $(0, -1, 4)$ and $(3, -2, 5)$ respectively.

    If $\overrightarrow{KM} = 3\overrightarrow{KL}$, find the coordinates of M.

    A $(1, -1, 3)$

    B $(1, 0, -1)$

    C $(9, -4, 7)$

    D $(9, -9, 27)$

15. $h(x) = \dfrac{4}{x^2 - 2x - 8}$.

    For what values of x is $h(x)$ undefined?

    A -2 and 4

    B -1 and 2

    C   0 and 4

    D   2 and -4

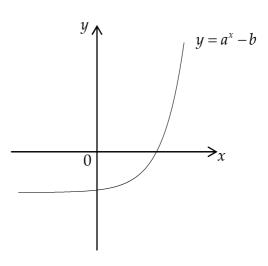16. Here are two statements about the graph with equation $y = a^x - b$, shown opposite.

(1) $0 < a < 1$;
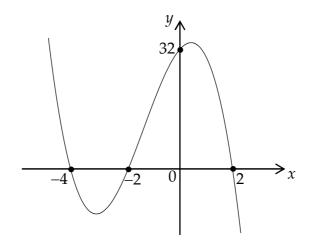
(2) y is always increasing

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

17. The diagram shows part of the graph of a cubic.
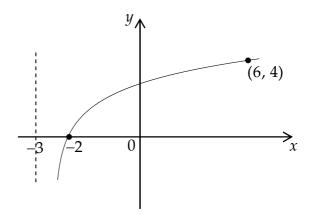

What is the equation of this graph?

A $y = -32(x+2)(x-2)(x-4)$

B $y = (x+2)(x-2)(x-4)$

C $y = -2(x-2)(x+2)(x+4)$

D $y = 4(x-2)(x+2)(x+4)$

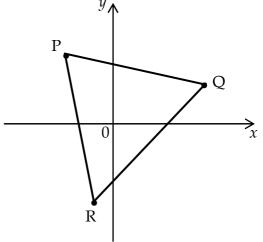18. Given that $\log_4 y = 2 - \log_4 5x$, express y in terms of x.

    A $y = \dfrac{2}{5x}$

    B $y = \dfrac{16}{5x}$

    C $y = \dfrac{1}{25x^2}$

    D $y = 25x^2$

19. If $\boldsymbol{p} \cdot (\boldsymbol{p} - \boldsymbol{q}) = 18$ and $\left|\boldsymbol{p}\right| = 3$, find the value of $\boldsymbol{p} \cdot \boldsymbol{q}$

    A   −15

    B   −9

    C   2

    D   15

20. The diagram shows part of the curve with equation $y = p\log_3(x + k)$.

    What is the value of p?

    A   −3

    B   −2

    C   2

    D   4

End of Section A

21. Triangle PQR has vertices P(−3, 5), Q(7, 3) and R(−1, −5), as shown.

(*a*) Find the equation of the median RM. **3**

(*b*) Find the equation of the altitude AP. **3**

(*c*) Find the coordinates of the point of intersection of RM and AP. **2**

22. Find the stationary points on the curve given by $y = x^3 - 9x^2 + 24x - 2$ and determine their nature. **7**

23. (*a*) Functions f and g are defined on suitable domains by

$$f(x) = 2x^2 + 5 \text{ and } g(x) = x - 1$$

Find $f(g(x))$. **2**

(*b*) Sketch the curve with equation $y = f(g(x))$. **3**

24. (*a*) Show that $2\sin\left(x + \frac{\pi}{6}\right) - 2\cos x = \sqrt{3}\sin x - \cos x$. **2**

(*b*) Express $\sqrt{3}\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence, or otherwise, solve $2\sin\left(x + \frac{\pi}{6}\right) = 2\cos x + \sqrt{3}$, where $0 \le x \le 2\pi$. **4**

End of question paper

Mathematics	Time allowed	NATIONAL
Higher	1 hour 10 minutes	QUALIFICATIONS
Paper 2		
Practice Paper A		

Read carefully

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FORMULAE LIST

Circle:

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Scalar Product : $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

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Table of standard derivatives :

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Table of standard integrals :

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ALL questions should be attempted.

1.   K is the point $(3, -2, 3)$, L$(5, 0, 7)$ and M$(7, -3, -1)$.

   (*a*)   Write down the components of $\overrightarrow{KL}$ and $\overrightarrow{KM}$.   **2**

   (*b*)   Calculate the size of angle LKM.   **5**

2.   (*a*)   (i)   Show that $(x-2)$ is a factor of $f(x) = 2x^3 - 3x^2 - 3x + 2$.

      (ii)   Hence factorise $f(x)$ fully.   **4**

   (*b*)   Solve $2(x^3 + 1) = 3x(x+1)$.   **2**

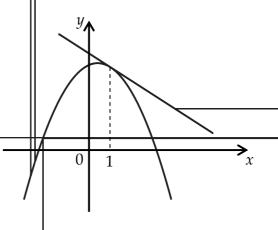3.   (*a*)   Find the equation of the tangent
           to the parabola with equation

           $$y = 6 + x - x^2$$

           at the point (1, 6).

                                                                    4

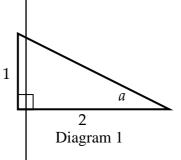   (*b*)   Show that this line is also a tangent to the circle with equation

           $$x^2 + y^2 - 4x - 14y + 51 = 0$$ **5**

4.   In the right-angled triangle shown in
     Diagram 1, $\tan a = \frac{1}{2}$.

   (*a*)   Find the exact values of

      (i) $\cos a$;

      (ii) $\cos 2a$.   **3**
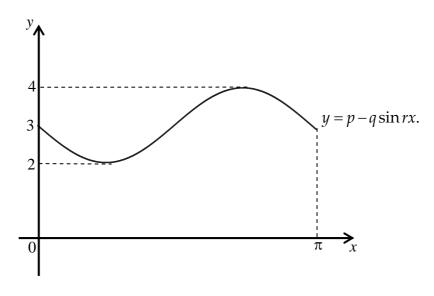

Diagram 1

   In the right-angled triangle shown in

   Diagram 2, $\tan b = \frac{1}{3}$.

   (*b*)   Find the exact value of $\sin(2a + b)$.   **5**

Diagram 2

Page 3

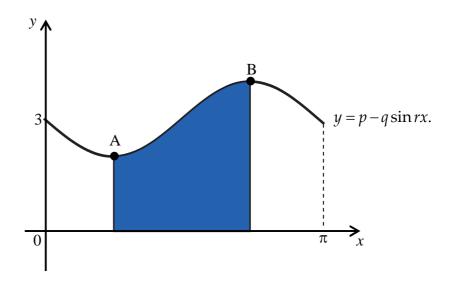**5.** Solve $\log_9(x+2) = \dfrac{1}{2} + \log_9(x-5)$, $x > 5$. **5**

6. The diagram below shows part of the graph of $y = p - q\sin rx$.

(a) Write down the values of p, q and r. **3**

The graph of $y = p - q\sin rx$. has a minimum turning point at A and a maximum turning point at B.

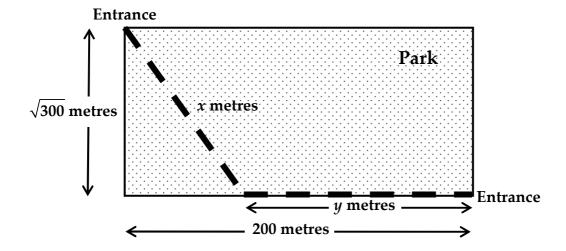*(b)* Calculate the shaded area in the diagram above. **7**

7. Cobalt-60 is used in food irradiation and decays to Nickel-60, which is a stable substance.

Cobalt-60 decays according to the law $m_t = m_0 e^{kt}$, where m_0 is the initial mass of Cobalt-60 present and m_t is the mass remaining after t years.

The time taken for half the mass of Cobalt-60 to decay to Nickel-60 is 5 years.

(*a*) Find the value of *k*, giving your answer correct to 3 significant figures. **3**

(*b*) In a sample of Cobalt-60 what percentage has decayed to Nickel-60 after 2 years? **3**

8. A rectangular park measures 200 metres by $\sqrt{300}$ metres.

A path connecting the two entrances, at opposite corners of the park, is to be laid through the park as shown.

The cost per metre of laying the path through the park is twice the cost, per metre, of laying the path along the perimeter.

(*a*) Show that the total cost of laying this path can be modelled by

$$C(x) = 2x + 200 - \sqrt{x^2 - 300}$$

3

(*b*) Find the value of *x* which would minimise the cost of laying the path. **6**

End of Question Paper

Paper 1

Section A

1.	D	11.	D
2.	D	12.	A
3.	C	13.	C
4.	A	14.	C
5.	B	15.	A
6.	A	16.	C
7.	D	17.	C
8.	D	18.	B
9.	A	19.	B
10.	D	20.	C

Section B

21. (*a*) $3x - y - 2 = 0$

    (*b*) $x + y - 2 = 0$

    (*c*) (1, 1)

22. Maximum turning point at (2, 18)

    Minimum turning point at (4, 14)

23. (*a*) $2(x-1)^2 + 5$ or $2x^2 - 4x + 7$

    (*b*)

24. (*a*) Proof

    (*b*) $2\sin\left(x - \frac{\pi}{6}\right)$

    (*c*) $\left\{\frac{\pi}{2}, \frac{5\pi}{6}\right\}$

Paper 2

1. (a) $\overrightarrow{KL} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ and $\overrightarrow{KM} = \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$

    (b) $110 \cdot 8°$ or $1 \cdot 934$ radians

2. (a) (i) Show that $f(2) = 0$ (ii) $f(x) = (x+1)(x-2)(2x-1)$

    (b) $\left\{ -1, \frac{1}{2}, 2 \right\}$

3. (a) $x + y - 7 = 0$

    (b) Proof

4. (a) (i) $\cos a = \dfrac{1}{\sqrt{5}}$ (ii) $\cos 2a = -\dfrac{3}{5}$

    (b) $\sin(2a+b) = \dfrac{9}{5\sqrt{10}}$

5. $x = \dfrac{17}{2}$

6. (a) $p = 3$, $q = 1$ and $r = 2$

    (b) $\dfrac{3\pi}{2} \approx 4 \cdot 712$

7. (a) $k = -0 \cdot 139$

    (b) $24 \cdot 1\%$

8. (a) Proof

    (b) $x = 20$