H

Read carefully

Calculators may NOT be used in this paper

Section A – Questions 1 – 20 (40 marks)

HB pencil

Section B (30 marks).

Read Carefully

[illegible] **Mathematics Higher (Section A)** [illegible]

[illegible] **HB pencil** [illegible]

[illegible] **your name**, **date of birth**, **SCN** [illegible]

[illegible]

[illegible]

[illegible]

[illegible] **only one correct** [illegible]

[illegible] **not** [illegible]

[illegible] **answer sheet for Section A inside the front cover of your answer book** [illegible]

Sample Question

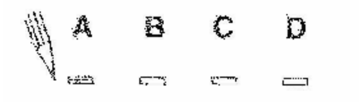
[illegible] $y = x^{[illegible]} - [illegible]x$ [illegible]

[illegible] $x =$ [illegible]

[illegible]

[illegible] $-$ [illegible]

[illegible] **A** [illegible] **A** [illegible] **pencil** [illegible]



Changing an answer

[illegible] **D** [illegible]

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted.

[illegible] $\begin{pmatrix} \square \\ \square \\ -\square \end{pmatrix}$ [illegible] $\begin{pmatrix} -\square \\ \square \\ \square \end{pmatrix}$ [illegible]

[illegible] $\overrightarrow{\square\square}$ [illegible]

(A) $\sqrt{\square}$

(B) $\sqrt{\square\square}$

(C) $\sqrt{\square\square}$

(D) $\sqrt{\square\square}$

[illegible] $f(x) = x^{\square} - \square x + \square$ [illegible] $f'(-\square)$ [illegible]

(A) [illegible]

(B) [illegible]

(C) [illegible]

(D) [illegible]

[illegible] $\int \left(x^{\frac{\square}{\square}} - x^{-\square} \right) dx$

(A) $\frac{\square}{\square} x^{-\frac{\square}{\square}} + \square x^{-\square} + c$

(B) $x^{\frac{\square}{\square}} + x^{-\square} + c$

(C) $x^{\frac{\square}{\square}} - x^{-\square} + c$

(D) $\frac{\square}{\square} x^{\frac{\square}{\square}} + x^{-\square} + c$

□□ □ f □ $f(x)=□x+□$

□ $f(f(x))$

A. $f(f(x))=□x+□$

B. $f(f(x))=□x+□$

C. $f(f(x))=□x^{□}+□x+□$

D. $f(f(x))=□x^{□}+□x+□$

□□ □ $□\sqrt{□}$ □ $\frac{\pi}{□}$ □ $\frac{□\pi}{□}$

A. $-□$

B. $-□$

C. $□\sqrt{□}$

D. $□\sqrt{□}$

□□ □ $(□,-□)$ □ $(□,-□)$

A. $(x-□)^{□}+(y+□)^{□}=□$

B. $(x-□)^{□}+(y+□)^{□}=□$

C. $(x+□)^{□}+(y-□)^{□}=□$

D. $(x+□)^{□}+(y-□)^{□}=□$

□□ $f(x)=□x^{□}-x^{□}-□x+□$

□ $f(x)$ □ $x+□$

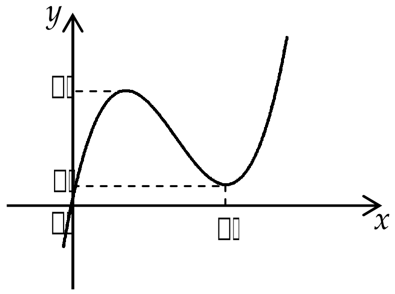
A. $-□$

B. $-□$

C. □

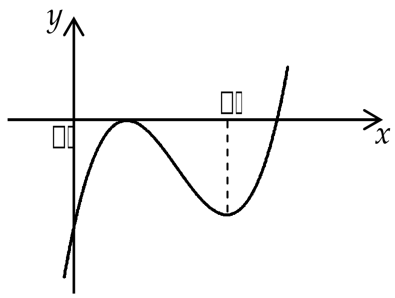
D. □

[illegible] $y = f(x)$ [illegible]

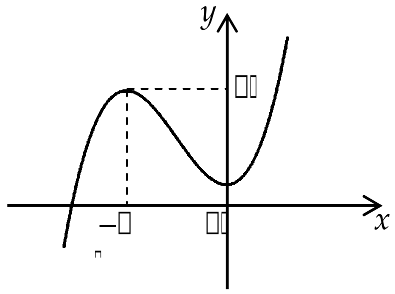


[illegible] $y = -f(x)$ [illegible]

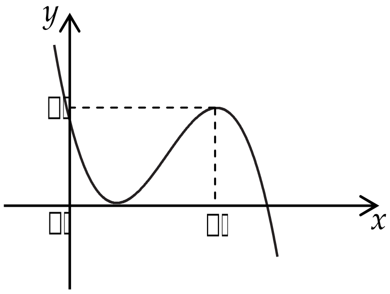
[illegible]



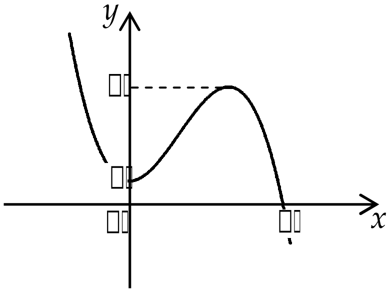
[illegible]



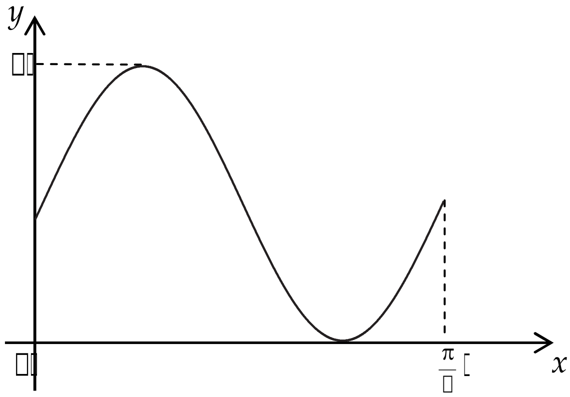
[illegible]



[illegible]



[illegible] $y = p + [illegible]\,qx$ [illegible] p [illegible] q



	p	q
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible] $u_{n+[illegible]} = [illegible] - [illegible]u_n$ [illegible]

[illegible] $u_{[illegible]} =$ [illegible] $u_{[illegible]}$ [illegible]

[illegible]

[illegible] k [illegible] $kx^{[illegible]} - [illegible]x + [illegible] = [illegible]$ [illegible]

[illegible]

[illegible] $\int [illegible]x + [illegible]\; dx$

[illegible] $[illegible]x + [illegible] + c$

[illegible] $[illegible]x^{[illegible]} + [illegible]x + c$

[illegible] $[illegible]x + [illegible] + c$

[illegible] $[illegible]x^{[illegible]} + [illegible]x + c$

□□□ □□□□□□□□□□ $f'(□x□)=□x^{□}$ □□□□□ $f(□□)=□$ □□□□□□□□□□□ □ □□□□□□ $f(x)$ □□□□□□□ □□□□□x□□

□ □□ $f(x)=□□x-□$ □

□ □□ $f(x)=□x^{□}-□$ □

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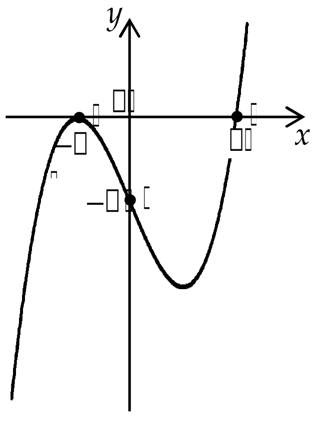

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□ □□ $y=-□□x-□□□^{□}□x+□□□$

□ □□ $y=-□x+□□□^{□}□x-□□□$

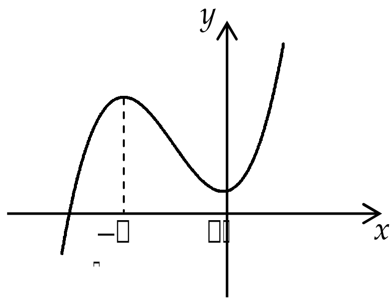
□ □□ $y=□x-□□□^{□}□x+□□□$

□ □□ $y=□□x+□□□^{□}□x-□□□$

□ □

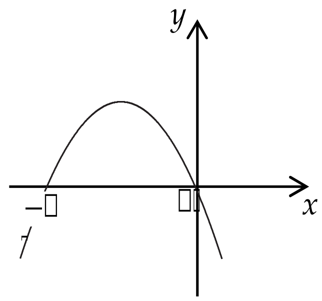
[illegible] $y=f(x)$ [illegible]

[illegible] $x=$ [illegible] $x=-$ [illegible]

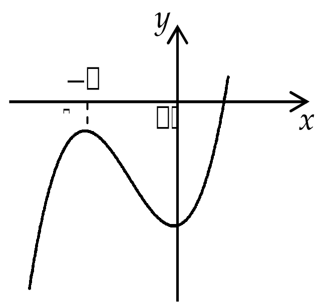


[illegible] $y=f'(x)$ [illegible]

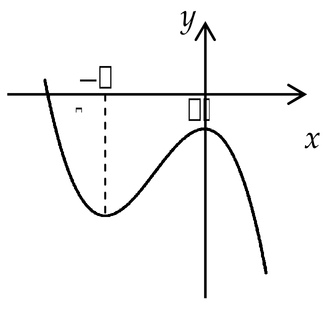
[illegible]



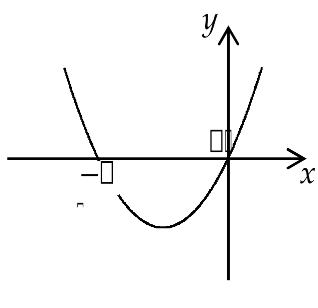
[illegible]



[illegible]



[illegible]



[illegible] $x^{[illegible]} + [illegible]x -$ [illegible] $(x + p)^{[illegible]} + q$ [illegible] q

[illegible] $t -$ [illegible] $=$ [illegible] t

[illegible] $p = [illegible]x^{-[illegible]}$ [illegible] p [illegible] x [illegible] $x =$ [illegible]

[illegible] $- [illegible]x - x^{[illegible]} <$ [illegible]

(A) $-[illegible] < x < [illegible]$

(B) $x < -[illegible]$ [illegible] $x > [illegible]$

(C) $x < -[illegible]$ [illegible] $x > [illegible]$

(D) $-[illegible] < x < [illegible]$

End of Section A

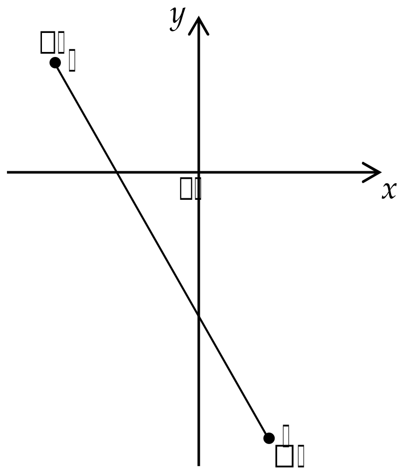
[illegible]

SECTION B

ALL questions should be attempted.

Marks

21. [illegible] y [illegible] $-$ [illegible]



4

22. [illegible] $y =$ [illegible] $x +$ [illegible] $x^{\square} + y^{\square} -$ [illegible] $x -$ [illegible] $y -$ [illegible] $=$ [illegible] 6

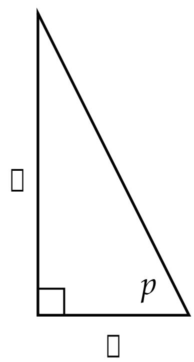
23. [illegible] p

(*a*) [illegible]

[illegible] p

[illegible] p 3

(*b*) [illegible] $p =$ [illegible] $p + p$ [illegible] p 4



24. [illegible] f [illegible] f [illegible] x [illegible] $= x^{\square} -$ [illegible] $x^{\square} -$ [illegible] $x +$ [illegible] $\leq x \leq$ [illegible]

[illegible] f 5

Marks

25. (*a*) [illegible] $\sqrt{}$ [illegible] $x^\circ - \sqrt{}$ [illegible] x° [illegible] k [illegible] $x - a)^\circ$ [illegible] $k >$ [illegible]

[illegible] $\leq a <$ [illegible] **4**

(*b*) [illegible]

[illegible] $+ \sqrt{}$ [illegible] $x^\circ - \sqrt{}$ [illegible] x° [illegible]

[illegible] x [illegible] $\leq x <$ [illegible] **4**

End of question paper

H

Mathematics
Higher
Paper 2
Practice Paper E

Time allowed
1 hour 10 minutes

NATIONAL QUALIFICATIONS

Read carefully

1 **Calculators may be used in this paper**.

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

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or $\boldsymbol{a} \,.\, \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
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Table of standard derivatives :

$f(x)$	$f'(x)$
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Table of standard integrals :

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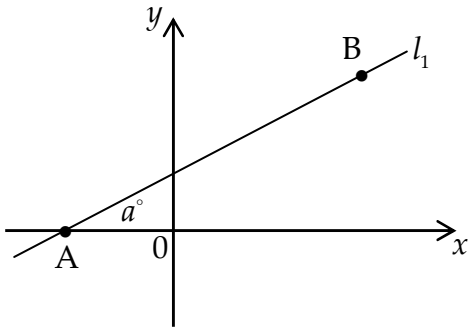
ALL questions should be attempted.

Marks

1. (*a*) A line, l_1, passes through the points $A(-3, 0)$ and $B(5, 4)$.

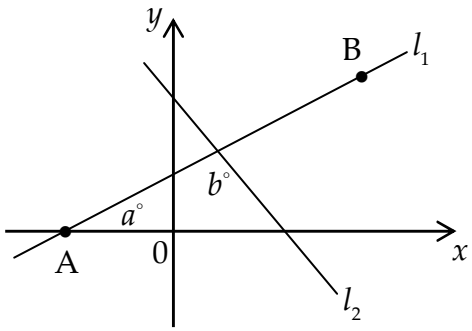
The line makes an angle of a° with the positive direction on the x-axis.



Find the value of a. **3**

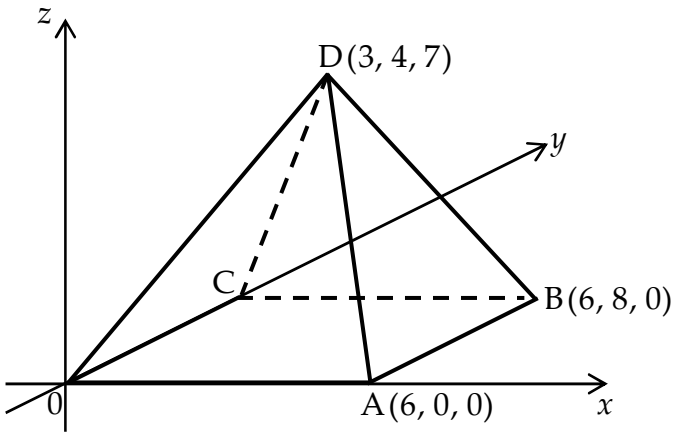
(*b*) A second line, l_2, with equation $4x + 3y = 12$, crosses the line in (*a*).

The angle between the two lines is b°, as shown.



Find the value of b. **4**

2. The rectangular based pyramid D,OABC has vertices $A(6, 0, 0)$, $B(6, 8, 0)$ and $D(3, 4, 7)$.



(*a*) (i) Write down the coordinates of C.

(ii) Express $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in component form. **3**

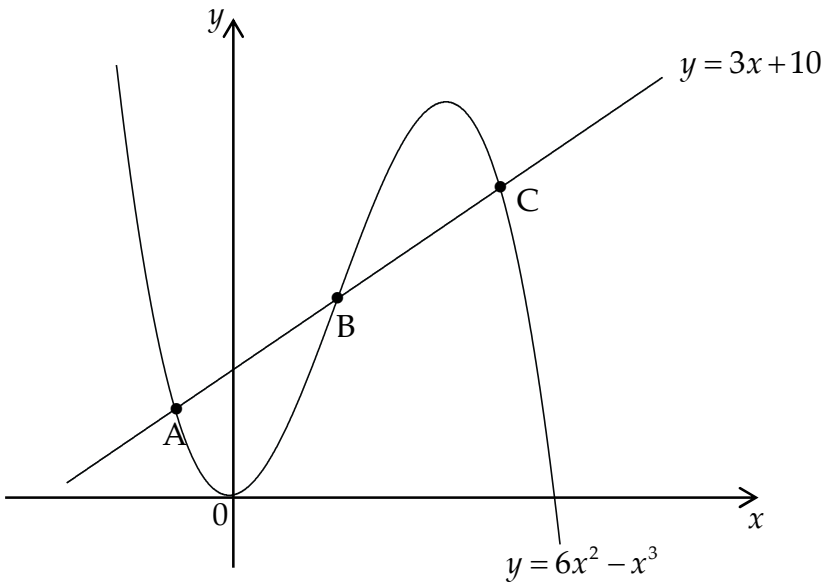
(*b*) Calculate the size of angle CAD. **5**

Marks

3. (*a*) (i) Show that $(x-2)$ is a factor of $x^3-6x^2+3x+10$.

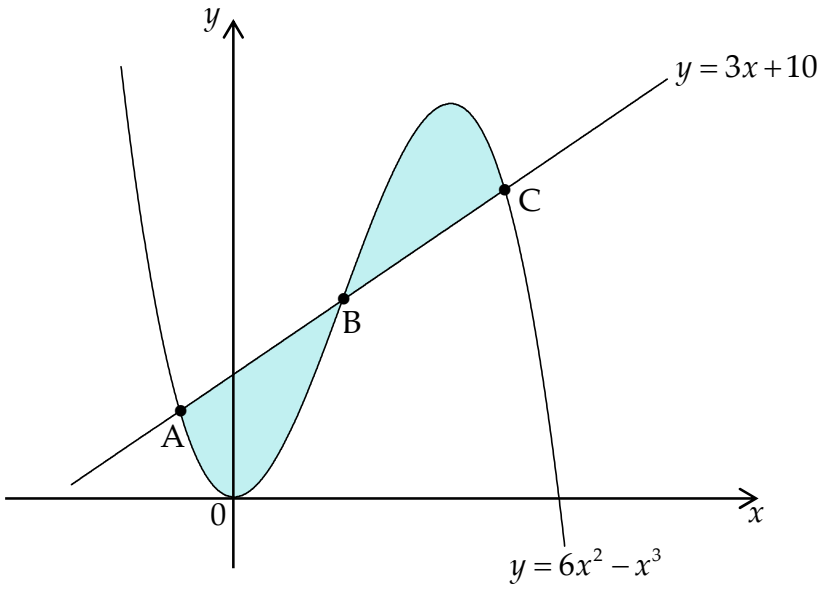
(ii) Hence factorise $x^3-6x^2+3x+10$ fully. **4**

The line with equation $y=3x+10$ intersects the curve with equation $y=6x^2-x^3$ at the points A, B and C.



(*b*) Find the x-coordinates of the points A and C. **3**

The area between the curve and the line from A to C is shaded in the diagram below.



(*c*) Calculate the total shaded area shown in the diagram. **7**

Marks

4. Solve $2\cos 2x - \sin x + 1 = 0$ for $0 \le x < 2\pi$. **6**

5. A new '24 hour anti-biotic' is being tested on a patient in hospital.

It is know, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.

On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(*a*) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? **1**

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(*b*) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.

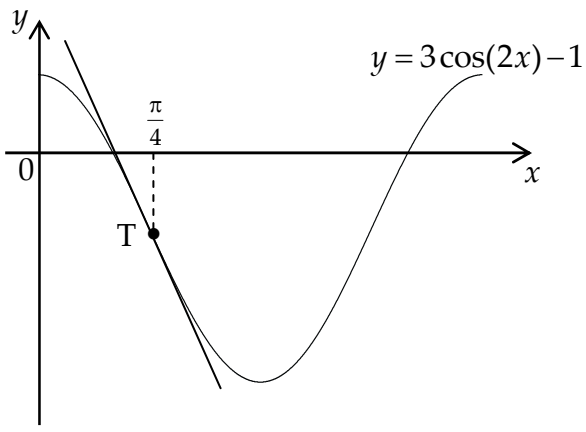
Write down the values of *a* and *b*. **2**

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(*c*) Is it safe to administer this anti-biotic over an extended period of time? **4**

6. The diagram shows part of the graph of $y = 3\cos(2x) - 1$.

Find the equation of the tangent at the point T, where $x = \frac{\pi}{4}$.



7

7. Solve $\log_x(x+2) + \log_x(2x-3) = 2,\ x > \frac{3}{2}$. **5**

Marks

8. A circle has the following properties:

- The x-axis and the line $y = 20$ are tangents to the circle.
- The circle passes through the points $(0,2)$ and $(0,18)$.
- The centre lies in the first quadrant.

Find the equation of this circle. **6**

End of Question Paper

Paper 1

Section A

1.	C	11.	C
2.	B	12.	C
3.	D	13.	D
4.	B	14.	C
5.	B	15.	D
6.	C	16.	D
7.	A	17.	A
8.	C	18.	B
9.	C	19.	C
10.	A	20.	C

Section B

21. $3x - 5y - 7 = 0$

22. Point of contact $(-3, 4)$

23. (*a*) (i) $\frac{1}{\sqrt{5}}$ (ii) $-\frac{3}{5}$

(*b*) $-\frac{11}{5\sqrt{5}}$

24. Maximum value 1, minimum value -7

25. (*a*) $4\cos(x - 315)^\circ$

(*b*) Maximum value 7 at $x = 315$

Paper 2

1. (*a*) $a = 26 \cdot 6$
   (*b*) $b = 100 \cdot 3$

2. (*a*) (i) C(0, 6, 0) (ii) $\overrightarrow{AC} = \begin{pmatrix} -6 \\ 6 \\ 0 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix}$
   (*b*) $54 \cdot 9^\circ$ or $0 \cdot 958$ radians

3. (*a*) (i) Proof (ii) $(x-2)(x-5)(x+1)$
   (*b*) A : $x = -1$ B : $x = 5$
   (*c*) $\frac{81}{2}$ square units

4. $\left\{0 \cdot 848,\ 2 \cdot 294,\ 4 \cdot 712\left(\frac{3\pi}{2}\right)\right\}$

5. (*a*) $50\,mg$
   (*b*) $u_{n+1} = 0 \cdot 2u_n + 250$
   (*c*) $312 \cdot 5 < 350 \Rightarrow$ safe to administer long term

6. $6x + y - \frac{3\pi}{2} - 1 = 0$

7. $x = 2$

8. $(x-6)^2 + (y-10)^2 = 100$