Mathematics

Mathematics 1

Intermediate 2

4883

Spring 1999

HIGHER STILL

Mathematics

Mathematics 1
Intermediate 2

Support Materials

INTRODUCTION

These support materials for Mathematics were developed as part of the Higher Still Development Programme in response to needs identified at needs analysis meetings and national seminars.

Advice on learning and teaching may be found in *Achievement for All* (SOEID 1996), *Effective Learning and Teaching in Mathematics* (SOEID 1993) and in the Mathematics Subject Guide.

This support package provides student material to cover the content of Mathematics 1 within the Intermediate 2 course. The depth of treatment is therefore more than is required to demonstrate competence in the unit assessment; that is, it goes beyond minimum grade C.

The content of Mathematics 1 (Int 2) is set out in the landscape pages of content in the Arrangements document where the requirements of the unit Mathematics 1 (Int 2) are also stated. Students are likely to have met some of the materials of this unit before, percentage work, volumes of solids, equations of lines and some of the algebraic work, though factorisation and arc and sector work will be new.

The material is designed to be directed by the teacher/lecturer, who will decide on the ways of introducing topics and in the use of exercises for consolidation and for formative assessment. The use of a scientific calculator would be helpful for some of the appreciation/depreciation work, though a basic calculator would generally suffice. Students should be encouraged to set down all working and, where appropriate, use mental calculations. Computers could be an advantage in the generation of some of the lines.

An attempt has been made to have the ‘easy’ questions at the start of each exercise, leading to more testing questions towards the end of the exercise. While students may tackle most of the questions individually, there are opportunities for collaborative working. Staff may wish to discuss points raised with individuals, groups and the whole class.

The specimen assessment questions at the end of the package are **not** intended to be only at minimum grade C. The National Assessment Bank packages for Mathematics 1 (Int 2) contain questions that meet the requirements of this unit.

This package gives opportunities to practise core skills, particularly the components of the Numeracy core skill, Using Number and Using Graphical Information, and Problem Solving. Information on the core skills embedded in the unit, Mathematics 1 (Int 2) and in the Intermediate 2 course is given in the final version of the Arrangements document. General advice and details of the Core Skills Framework can be found in the Core Skills Manual (HSDU June 1998).

Brief notes of advice on the teaching of each topic are given.

Format of Student Material

- Exercises on Calculations Involving Percentages
  Checkup for Calculations Involving Percentages

- Exercises on Volumes of Solids
  Checkup for Volumes of Solids

- Exercises on Linear Relationships
  Checkup for Linear Relationships

- Exercises on Algebraic Operations
  Checkup for Algebraic Operations

- Exercises on Properties of a Circle
  Checkup for Properties of a Circle

- Specimen Assessment Questions

- Answers for all exercises

CALCULATIONS INVOLVING PERCENTAGES

Revision

Students should find the first exercise quite straight-forward.

It involves: calculating percentages of quantities, simple interest and expressing one quantity as a percentage of another.

The following simple examples could be used as an introduction:

Example 1 Find 84% of £22 Show working to arrive at ... Ans £18·48

Example 2 At a ceilidh, 62·5% of the 80 people attending were female.
How many males were there?

Show working for 62·5% of 80 Ans 50
Therefore ... 30 male

Alternatively: find (100 – 62·5)% of 80 = 30 male

Example 3 Calculate the Simple Interest on £3200 for 3 months at 5% p.a.
* Explain 'p.a.' 'per annum' 'annually' etc. ...

Ans.	Interest for 1 year	= 5% of £3200	=	£160
	Interest for 1 month	= £160 ÷ 12	=	£13·33333...
	Interest for 3 month	= £13·33333.. . x 3	=	£40

Example 4 When buying a £650 cooker, I am asked for a deposit of £97·50.
What percentage deposit is this?

Ans. ($^{97·50}/_{650}$) x 100 = 15%

Exercise 1 may now be attempted.

A. Compound Interest

The difference between 'Simple' and 'Compound' Interest should be explained to students.

The terms 'Deposit', 'Rate of Interest' and how the rate rises and falls, 'Principal' and 'Amount' should be discussed with the students.

The following examples could be used:

Example 1 Mrs Paton deposits £400 in a bank and leaves it there for three years to gain compound interest at 5% per annum.

Calculate: (a) How much is in her account after 3 years.
(b) How much interest she gained.

Ans. (a)	Yr. 1 Interest = 5% of £400 = £20	Now in Bank £420
	Yr. 2 Interest = 5% of £420 = £21	Now in Bank £441
	Yr. 3 Interest = 5% of £441 = £22·05	Now in Bank £463·05
Ans. (b)	Total Interest gained = £463·05 – £400 = £63·05	

Most examples have been chosen so that the interest gained at the end of the year works out to complete pounds, making the calculation for the following year simple.
But that is not always the case in reality ...

Example 2 Mrs. Seaton deposits £430 in a bank and leaves it there for three years to gain compound interest at 5% per annum.

Calculate how much is in her account after 3 years.

Ans. Yr. 1 Interest = 5% of £430 = £21·50 Now in Bank £451·50

Yr. 2 Interest = 5% of £451 = £22·55

* interest worked out on complete pounds only

Now in Bank £451·50 + £22·55
= £474·05

Yr. 3 Interest = 5% of £474* = £23·70

*notice complete pounds again

Now in Bank £474·05 + £23·70
= $\underline{\underline{\text{£497·75}}}$

In such cases where the calculation has to be done over a large number of years the $\boxed{y^x}$ key on the calculator could be used.

Example 3 Calculate the compound interest on £4600 for 10 years at 6% p.a.

Ans. Amount in bank after 10 years = £4600 x 1·06 $\boxed{y^x}$ 10 = £8237·90

Interest gained = £8237·90 – £4600 = $\underline{\underline{\text{£3637·90}}}$

Exercise 2 may now be attempted.

B. Appreciation and Depreciation

The terms ‘appreciation’ and ‘depreciation’ should be explained to the students.

Example 1 I buy a flat for £40 000. In each of the following 3 years its value appreciated by 8%.

How much is the flat now worth after the 3 years?

Ans Yr. 1 apprec. 8% of £40 000 = £3200 Flat worth £43 200
Yr. 2 apprec. 8% of £43 200 = £3456 Flat worth £46 656
Yr. 3 apprec. 8% of £46 656 = £3732·48 Flat worth $\underline{\underline{\text{£50 388·48}}}$

It should be explained that the rates may change – rise or fall. They do not always remain constant.

Example 2 Tiger buys a new set of golf clubs for £600.

The clubs lose 5% of their value during the first year and 10% during the second year. How much are they worth after 2 years?

Ans	Yr. 1 deprec.	5% of £600 = £30	Clubs worth £570
	Yr. 2 deprec.	10% of £570 = £57	Clubs worth £513

Percentage Appreciation / Depreciation

Example 3 Arnold's clubs are worth £600 when new.
5 years later he sells them for £480

What is the percentage depreciation in the value of the clubs?

Ans Depreciation = £600 – £480
= £120

% Depreciation = $^{120}/_{600}$ x 100 = 20%

Exercise 3 may now be attempted. Q12 as extension only

C. Significant Figures (often written as **sig. figs.**)

The number of significant figures can be used to express the accuracy of a number or a measurement.

Significant Figures could be explained as follows:

For numbers greater than 1

- to round to 1 sig. fig. – round to the highest place value
- to round to 2 sig. figs. – round to the second highest place value
- to round to 3 sig. figs. – round to the third highest place value

For example :- the number **4269** ... the '**4**' is the highest place value,
the '**2**' second place value and so on.

4269 becomes 4000 to 1 sig. fig. 4300 to 2 sig. figs. 4270 to 3 sig. figs.

Exercise 4 may now be attempted.

Percentage Calculations rounded to a required number of Significant Figures

The exercise on this topic is similar to Exercise 2 and Exercise 3.

This time though, the answers are not as precise and rounding to a required number of significant figures is required. One board example should suffice.

Example Albert deposits £400 for 3 years in his Investment Account at a rate of 5% in year 1, 10% in year 2 and 8% in year 3.

How much will he have in the account after the 3 years?

Give your answer correct to 3 sig. figs.

Ans	Yr. 1 Interest	5% of £400 = £20	£420 in account
	Yr. 2 Interest	10% of £420 = £42	£462 in account
	Yr. 3 Interest	8% of £462 = £36·96	£498·96 in account

£499 in account (answer correct to 3 sig. figs.)

Exercise 5 may now be attempted.

The Checkup Exercise may now also be attempted.

VOLUMES OF SOLIDS

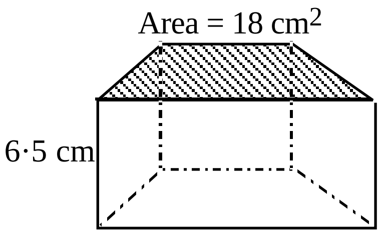
Students should be clear as to what a prism is.

Its volume should be defined:

$$\text{Volume}_{\text{prism}} = \text{Area}_{\text{base}} \times \text{height}$$

or $V = A \times h$

Example 1:

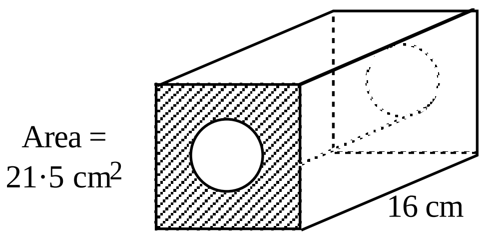


$V = A_{\text{base}} \times h$

$V = 18 \times 6{\cdot}5$

$V = 117 \text{ cm}^3$

Example 2 :



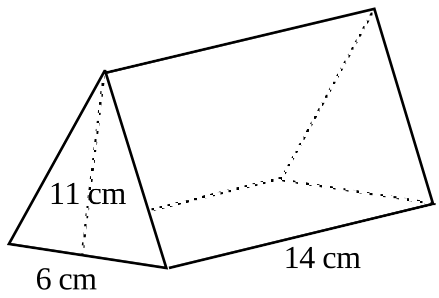
$V = A_{\text{base}} \times h$

$V = 21{\cdot}5 \times 16$

$V = 344 \text{ cm}^3$

It should be stressed that the ‘base’ need not be on the ‘bottom’ of the prism.

Example 3: *



Area $_{\text{end face}}$ = $^1/_2$ b x h

A = $^1/_2$ x 6 x 11

A = 33 cm^2

V = A $_{\text{base}}$ x h

V = 33 x 14

V = 462 cm^3

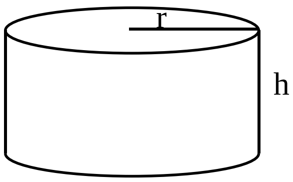
* - the two steps to solving these type of examples should be emphasised to the students.

Exercise 1, questions 1 and 2, should now be attempted.

Volume of a cylinder: Define a cylinder as a circular based prism.

V = A $_{\text{base}}$ x h

$\boxed{V = \pi r^2 h}$

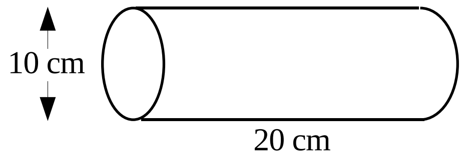


Example: Find the volume of this cylinder

V = πr^2h

V = 3·14 x 5 x 5 x 20

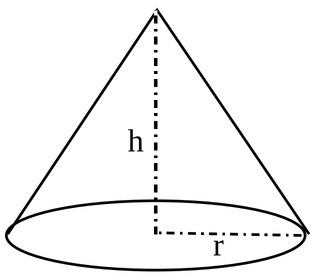
V = 1570 cm^3



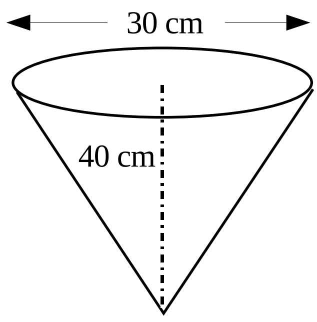
Exercise 1, questions 3 to 9, may now be attempted.

Students should be given the formula for the volume of a cone

$\boxed{V = {}^1/_3 \pi r^2 h}$



Example: Calculate the volume of this cone.



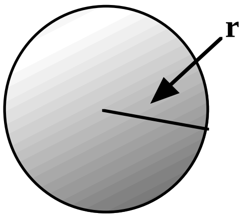
V = $^1/_3 \pi r^2$h

V = $^1/_3$ x 3·14 x 15 x 15 x 40

V = 9420 cm^3

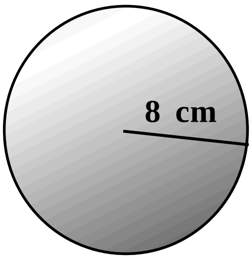
Exercise 2 may now be attempted.

Students should be given the formula for the volume of a sphere



$\boxed{V = {}^4/_3\pi r^3}$

Example 1: Calculate the volume of this sphere.

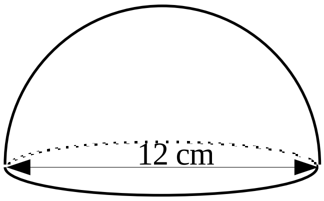


$V = {}^4/_3\pi r^3$

$V = 4 \times 3{\cdot}14 \times 8 \times 8 \times 8 \div 3$

$V = 2143{\cdot}6 \text{ cm}^3$

Example 2 Calculate the volume of this hemisphere :-



$V = {}^4/_3\pi r^3 \div 2$

$V = (4 \times 3{\cdot}14 \times 6 \times 6 \times 6 \div 3) \div 2$

$V = 452{\cdot}16 \text{ cm}^3$

Exercise 3 may now be attempted.

The Checkup Exercise may also be attempted.

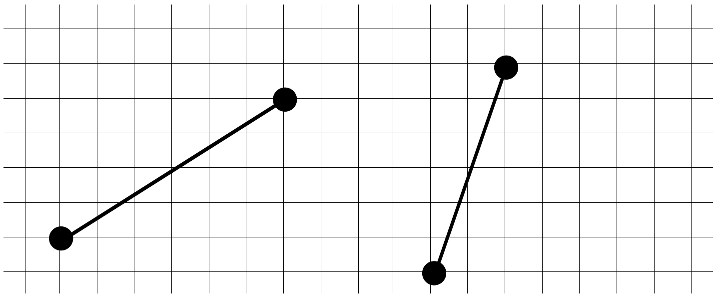
LINEAR RELATIONSHIPS

The gradient of a line (The use of graphics calculators may be used to enhance this topic)

A discussion should take place with the students as to the idea of the 'slope' or 'gradient' of a hill or road, leading to:

$$\boxed{\text{gradient} = \frac{\text{vertical distance}}{\text{horizontal distance}}}$$

Two simple examples should be used to show that the 'higher the gradient' –> the 'steeper the slope'.

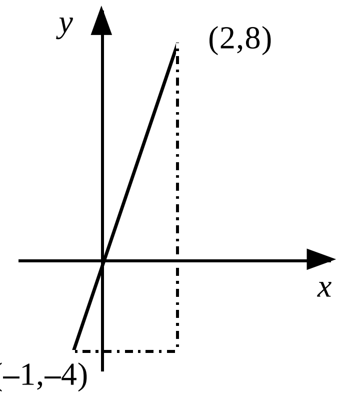


gradient = $^4/_6$ = $\mathbf{^2/_3}$ gradient = $^6/_2$ = **3**

Finding the gradient of a line in a coordinate diagram should be introduced leading to:

$$\boxed{\text{gradient} = \frac{y\text{ - shift}}{x\text{ - shift}}} \quad \text{or} \quad \boxed{\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}}$$

Example 1: Plot points (–1,–4) and (2,8) and find gradient of line joining them.



=> from diagram:

$$\text{gradient} = \frac{\text{vert}}{\text{horiz}} = \frac{12}{3} = 4$$

=> from formula:

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - (-4)}{2 - (-1)} = \frac{12}{3} = 4$$

Example 2: Find the gradient of the line joining (–6,5) to (2,–1).

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - (-6)} = \frac{-6}{8} = -^3/_4 \quad \text{(note the negative)}$$

This should lead to a discussion on the difference between a negative and a positive gradient.

Exercise 1 should now be attempted.

Sketching lines of the form $y = ax + b$.

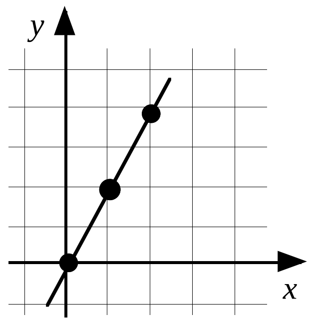
Students are likely to have met this topic before, either at General Level or possibly in Int 1.
The drawing of simple lines like: $y = 2x$, $y = 3x$, $y = {}^{1}/_{2}x$. should be revised
i.e. they each pass through the origin and have gradients 2, 3 and $^{1}/_{2}$.

Example 1: Line, $y = \mathbf{2}x$ =>

The gradient is **2**

=> from the origin, move 1 box right and **2** boxes upwards.

=> repeat etc.



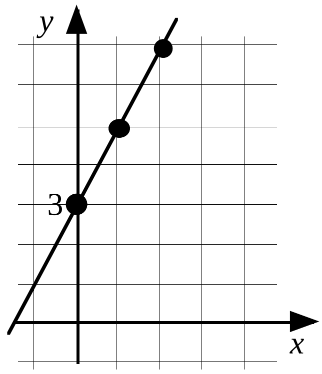
Example 2 : Line, $y = \mathbf{2}x + \underline{3}$ =>

The gradient is also **2**

=> the <u>3</u> tells us the line cuts the *y*-axis 3 units up from the origin.

=> from this point (0,3), move 1 box right and **2** boxes upwards.

=> repeat etc.



Students should then be shown how to make a quick sketch of any line $y = ax + b$ by:

(a) noting that the line cuts the *y*-axis at the point (0, <u>b</u>).

(b) finding a few more points by using the gradient

.

Three further examples:

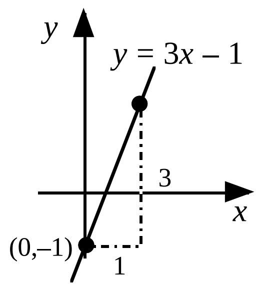
Example 3: Line, $y = \mathbf{3}x - \underline{1}$ =>

The gradient is **3**

=> the –<u>1</u> tells us the line cuts the *y*-axis 1 units down from the origin.

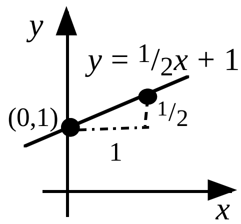
=> from this point (0,–1), move 1 box right and **3** boxes upwards.

=> repeat etc.



Example 4: Line, $y = {}^1/_2x + \underline{1}$ =>

The gradient is $^1/_2$



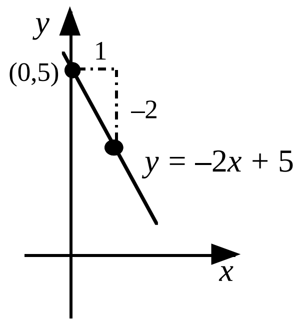
=> the +$\underline{1}$ tells us the line cuts the y-axis 1 units up from the origin.

=> from this point (0,1), move 1 box right and $^1/_2$ boxes upwards. (or 2 boxes right and 1 box up)

=> repeat etc.

Example 5: Line, $y = \mathbf{-2}x + \underline{5}$ =>

The gradient is **–2**



=> the +$\underline{5}$ tells us the line cuts the y-axis 5 units up from the origin.

=> from this point (0,5), move 1 box right and **2** boxes downwards.

=> repeat etc.

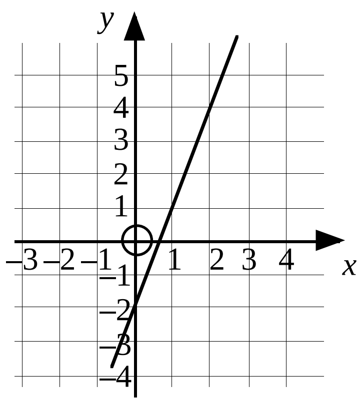
Exercise 2 may now be attempted.

Determining the equation of a straight line in the form $y = \text{a}x + \text{b}$ from its graph.

Students should be reminded that every line can be written in the form $y = \text{a}x + \text{b}$.
(There is no need to mention lines of form $x = \text{h}$ at this stage.)

3 examples which can be used in determining the equations of straight lines:

Example 1: <u>three</u> steps to finding its equation.



<u>Step 1:</u> Write down the general equation

$$y = \text{a}x + \text{b}$$

<u>Step 2:</u> look for where it cuts the y-axis
(0,–2) => b = –2

=> $y = \text{a}x - 2$

<u>Step 3:</u> Determine the gradient from any 2 points on the line (0,–2) & (1,1)

=> gradient = $\dfrac{1-(-2)}{1-0}$ = 3, (or find it by counting).

$$\boxed{\Rightarrow \quad y = 3x - 2}$$

Example 2:

<u>Step 1:</u> Write down the general equation

$y = ax + b$

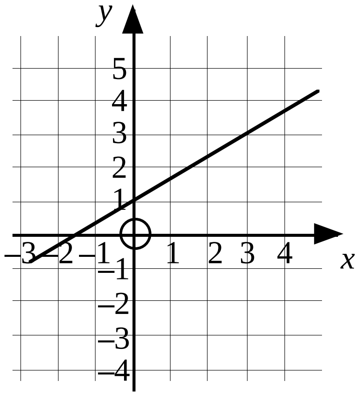
<u>Step 2:</u> look for where it cuts the y-axis
(0,1) => b = 1

=> $y = ax + 1$

<u>Step 3:</u> Determine the gradient from any 2 points on the line (0,1) & (3,3)

$$\Rightarrow \quad \text{gradient} = \frac{3-1}{3-0} = {}^2/_3$$

$$\boxed{\Rightarrow \quad y = {}^2/_3 x + 1}$$



Example 3:

<u>Step 1:</u> Write down the general equation

$y = ax + b$

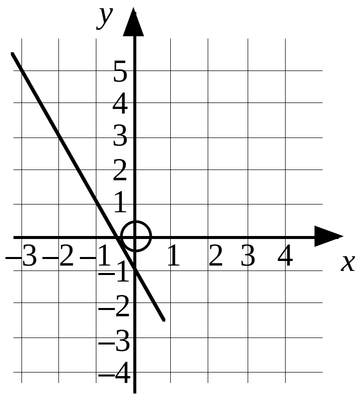
<u>Step 2:</u> look for where it cuts the y-axis
(0,–1) => b = –1

=> $y = ax - 1$

<u>Step 3:</u> Determine the gradient from any 2 points on the line (0,–1) & (–2,3)

$$\Rightarrow \quad \text{gradient} = \frac{3-(-1)}{-2-0} = -2$$

$$\boxed{\Rightarrow \quad y = -2x - 1}$$



Students should be reminded of the format of lines which are parallel to the x and y axes.

i.e. $x = a$ and $y = b$

Exercise 3 may now be attempted.

The Checkup Exercise may also be attempted.

ALGEBRAIC OPERATIONS

A. Multiplying Algebraic Expressions Involving Brackets

This is best illustrated by examples on board.

Example 1 Expand $5(x + 2)$

Ans. $5(x + 2)$
$= \underline{\underline{5x + 10}}$

Example 2 Multiply out $4(2p - 7)$

Ans. $4(2p - 7)$
$= \underline{\underline{8p - 28}}$

Example 3 Expand $x(3x + 6)$

Ans. $x(3x + 6)$
$= \underline{\underline{3x^2 + 6x}}$

Example 4 Multiply out $a(2a - 3m)$

Ans. $a(2a - 3m)$
$= \underline{\underline{2a^2 - 3am}}$

Exercise 1 may now be attempted.

There are various methods by which Double Brackets can be introduced.
Two are illustrated below:

Go over the following:

Method 1:

$$\begin{aligned} & (x + 2)(x + 3) \\ = \; & x(x + 3) + 2(x + 3) \\ = \; & x^2 + 3x + 2x + 6 \\ = \; & \underline{\underline{x^2 + 5x + 6}} \end{aligned}$$

Method 2:

$(x + 2)(x + 3)$

FOIL

F	F(irst)	$x \times x$	=	x^2
O	O(utside)	$x \times 3$	=	$3x$
I	I(nside)	$2 \times x$	=	$2x$
L	L(ast)	2×3	=	6

$x^2 + 3x + 2x + 6$
$= \underline{\underline{x^2 + 5x + 6}}$

The following examples can then be used to practice the chosen method:

Example 1 Multiply $(x + 1)(x + 4)$

Ans. $= \underline{\underline{x^2 + 5x + 4}}$

Example 2 Multiply $(x - 3)(x - 4)$

Ans. $= \underline{\underline{x^2 - 7x + 12}}$

contd.

Example 3 Multiply $(x - 1)(x + 2)$

Ans. $= \underline{\underline{x^2 + x - 2}}$

Example 4 Multiply $(2x + 7)(x - 3)$

Ans. $= \underline{\underline{2x^2 + x - 21}}$

Example 5 Multiply $(x + 3)^2$ Advise students to write this as $(x + 3)(x + 3)$ first.

Ans. $= \underline{\underline{x^2 + 6x + 9}}$

Exercise 2A may now be attempted.

More complicated expansion of brackets.

Use the following :

Example Multiply $(x + 3)(x^2 + 4x + 2)$

Ans.

$$
\begin{aligned}
&(x + 3)(x^2 + 4x + 2) \\
&= x(x^2 + 4x + 2) + 3(x^2 + 4x + 2) \\
&= x^3 + 4x^2 + 2x + 3x^2 + 12x + 6 \\
&= \underline{\underline{x^3 + 7x^2 + 14x + 6}}
\end{aligned}
$$

Exercise 2B may now be attempted.
Note that this exercise is appropriate to grades A/B

B. Factorising Algebraic Expressions

The Common Factor

After 'multiplying out' brackets, we now turn to 'putting into' brackets.
This process is called factorising.

For example: $2(x + 3) = 2x + 6$ so, in reverse $2x + 6 = 2(x + 3)$

2 is the highest factor of $2x$ and 6, so 2 goes outside the bracket.

Also $2a(a - 4) = 2a^2 - 8a$ so, in reverse $2a^2 - 8a = 2a(a - 4)$

$2a$ is the highest factor of $2a^2$ and $8a$, so $2a$ goes outside the bracket.
$a - 4$ is then required inside the bracket.

Answers should always be checked by multiplying out the factorised answer.

The following could be used to reinforce the work just done on factorising:

Example 1 Factorise $9x + 15$

Ans. 'What is the highest number to go into $9x$ and 15?' **3**

'Are there any letters common to $9x$ and 15?' **No**

So only 3 comes before a bracket. **3( )**

'What is required in the bracket so that the $9x$ can be found?' $\mathbf{3x}$

$$\mathbf{3(3x + \quad)}$$

'What is required in the bracket so that the 15 can be found?' **5**

$$\mathbf{3(3x + 5)}$$

$$9x + 15 = \underline{\underline{3(3x + 5)}}$$ Check by multiplying out

Example 2 Factorise $18w^2 - 12w$

Ans. 'What is the highest number to go into $18w^2$ and $12w$?' **6** (not 3)

'Are there any letters common to $18w^2$ and $12w$?' **Yes** $\boldsymbol{w}$

So $6w$ comes before a bracket. $\mathbf{6w(\quad)}$

'What is required in the bracket so that the $18w^2$ can be found?' $\mathbf{3w}$

$$\mathbf{6w(3w - \quad)}$$

'What is required in the bracket so that the $12w$ can be found?' **2**

$$\mathbf{6w(3w - 2)}$$

$$18w^2 - 12w = \underline{\underline{6w(3w - 2)}}$$ Check by multiplying out

Exercise 3 may now be attempted.

Difference of Two Squares

Go over the following:

Show that: $\mathbf{4}^2 - \mathbf{1}^2 = 16 - 1 = 15 = 3 \times 5 =$ $(\mathbf{4} - \mathbf{1})(\mathbf{4} + \mathbf{1})$

$\mathbf{5}^2 - \mathbf{2}^2 = 25 - 4 = 21 = 3 \times 7 =$ $(\mathbf{5} - \mathbf{2})(\mathbf{5} + \mathbf{2})$

Ask for response for $\mathbf{6}^2 - \mathbf{1}^2 =$

$\mathbf{6}^2 - \mathbf{2}^2 =$

$\mathbf{6}^2 - \mathbf{3}^2 =$

$\mathbf{8}^2 - \mathbf{2}^2 =$

Students recognise the pattern $\boldsymbol{a}^2 - \boldsymbol{b}^2 = (\boldsymbol{a} - \boldsymbol{b})(\boldsymbol{a} + \boldsymbol{b})$

The results should be checked by multiplying out $(a - b)(a + b)$

This result should now be illustrated with a range of examples. such as $w^2 – 9$, $x^2 – 4$, $b^2 – 25$. Once students are confident with these examples, the result for $a^2 – b^2$ can be extended to included examples such as $4x^2 – 25$ and $9y^2 – 4z^2$.

Example 1 Factorise $4x^2 – 25$

Ans. $4x^2 – 25$

$= (2x)^2 – (5)^2$

$= \underline{\underline{(2x – 5)(2x + 5)}}$

Example 2 Factorise $9y^2 – 4z^2$

Ans. $9y^2 – 4z^2$

$= (3y)^2 – (2z)^2$

$= \underline{\underline{(3y – 2z)(3y + 2z)}}$

Show students how to look out for and deal with factorising examples involving a common factor <u>and</u> the difference of two squares using the following two examples:

Example 3 Factorise fully $2x^2 – 32$

Ans. $2x^2 – 32$

common factor 2

$= 2(x^2 – 16)$

$= \underline{\underline{2(x – 4)(x + 4)}}$

Example 4 Factorise fully $kx^2 – 25ky^2$

Ans. $kx^2 – 25ky^2$

common factor k

$= k(x^2 – 25y^2)$

$= \underline{\underline{k(x – 5y)(x + 5y)}}$

Check by multiplying out

Exercise 4 may now be attempted.

Trinomial Expressions (Quadratic Expressions) Factorising $ax^2 + bx + c$

The terms: ‘trinomial’, ‘quadratic’ and ‘expression’ (as opposed to equation) should be explained.

TYPE 1 Factorising $ax^2 + bx + c$ with $a = 1$.

e.g. $x^2 + 4x + 3$ $x^2 – 4x – 5$ $x^2 + x – 12$

Illustrate: $(x + 2)(x + 3) = x^2 + 5x + 6$ so, in reverse $x^2 + 5x + 6 = (x + 2)(x + 3)$

This is one way of showing the reverse process:

Example Factorise $x^2 + 5x + 6$

Draw up a table.

x	1	6	2	3
x	6	1	3	2

The x’s are for the $x \times x = x^2$ term.

The numbers on the r.h.s. of the table are factors of the constant 6
(read vertically 1x6 6x1 2x3 3x2)

The middle term, the x term, has not been mentioned yet!

In turn, multiply diagonally, then add to look for that x term.

Here $6x + 1x = 7x$ No !

x	1	6	2	3
x	6	1	3	2

Here $2x + 3x = 5x$ Yes !

x	1	6	2	3
x	6	1	3	2

So $x^2 + 5x + 6$ can be factorised by looking at this table:

$$\begin{array}{c|cccc} x & 1 & 6 & 2 & 3 \\ \hline x & 6 & 1 & 3 & 2 \end{array}$$

Having settled for $\begin{array}{c|c} x & 2 \\ \hline x & 3 \end{array}$ we now bracket horizontally

$$x^2 + 5x + 6 \;=\; \underline{\underline{(x + 2)(x + 3)}}$$

$$\begin{array}{c|c} (x & 2) \\ \hline (x & 3) \end{array}$$

The method chosen should be reinforced with a number of examples:

Example 1 Factorise $x^2 - 8x + 7$

Ans Table $\begin{array}{c|cccc} x & 1 & 7 & -7 & -1 \\ \hline x & 7 & 1 & -1 & -7 \end{array}$

try $\begin{array}{c|c} x & 1 \\ \hline x & 7 \end{array}$ try $\begin{array}{c|c} x & -7 \\ \hline x & -1 \end{array}$

$7x + 1x = 8x$ No $-7x - 1x = -8x$ Yes

$x^2 - 8x + 7 \;=\; \underline{\underline{(x - 7)(x - 1)}}$ Check by multiplying out

Example 2 Factorise $x^2 + 5x - 6$

Ans Table $\begin{array}{c|cccc} x & -6 & -2 & -1 & -3 \\ \hline x & 1 & 3 & 6 & 2 \end{array}$

try $\begin{array}{c|c} x & -6 \\ \hline x & 1 \end{array}$ try $\begin{array}{c|c} x & -2 \\ \hline x & 3 \end{array}$

$-6x + 1x = -5x$ No $-2x + 3x = x$ No

try $\begin{array}{c|c} x & -1 \\ \hline x & 6 \end{array}$

$-1x + 6x = 5x$ Yes

$x^2 + 5x - 6 \;=\; \underline{\underline{(x - 1)(x + 6)}}$ Check by multiplying out

Students should become faster at this, and even begin to recognise which products to choose.

Exercise 5 may now be attempted.

TYPE 2 Factorising $ax^2 + bx + c$, $a > 1$.

e.g. $2x^2 + 7x + 3$ $6x^2 - 5x - 6$ $8x^2 - 28x + 12$

Illustrate: $(3x + 2)(x + 3) = 3x^2 + 11x + 6$ so, in reverse $3x^2 + 11x + 6 = (3x + 2)(x + 3)$

Again, the chosen method should be reinforced with a number of examples as follows:

Example 1. Factorise $3x^2 + 11x + 6$

Ans. $3x^2 + 11x + 6$

Table

$3x$	6	1	2	3
x	1	6	3	2

try

$3x$	6
x	1

$3x + 6x = 9x$ No

try

$3x$	1
x	6

$18x + 1x = 19x$ No

try

$3x$	2
x	3

$9x + 2x = 11x$ Yes

$3x^2 + 11x + 6 = \underline{\underline{(3x + 2)(x + 3)}}$ Check by multiplying out

Example 2. Factorise fully: (explain 'fully')

$6x^2 + 4x - 16$

Notice here that a common factor can be taken out.

Ans. $6x^2 + 12x - 16$

$= 2(3x^2 + 2x - 8)$ ... making the factorising easier

Table

$3x$	–8	–4	–2	–1
x	1	2	4	8

$3x$	–4
x	2

shows correct factors

$6x - 4x = 2x$ Yes

$6x^2 + 12x - 16 = \underline{\underline{2(3x - 4)(x + 2)}}$ Check by multiplying out

Exercise 6 may now be attempted.

Note that this exercise is appropriate to grades A/B

Then Exercise 7 (miscellaneous examples) should be attempted.

The Checkup Exercise may then also be attempted.

PROPERTIES OF A CIRCLE

Revision.

The terms Diameter, Radius, Circumference, Area of a circle should be revised along with the revision of circumference and area.
Some straightforward examples should be gone over with the students.

Example 1 Calculate the circumference of a circle with radius 5 cm.

Ans. $C = \pi d$
$= 3{\cdot}14 \times 10$
$= \underline{\underline{31{\cdot}4 \text{ cm}}}$

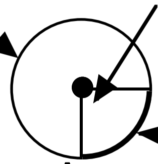
Example 2 Calculate the area of a circle with diameter 20 cm.

Ans. $A = \pi r^2$
$= 3{\cdot}14 \times 10 \times 10$
$= \underline{\underline{314 \text{ cm}^2}}$

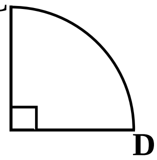
Note: some students may require further practice in finding the circumference and the area.

A. Length of the ARC of a Circle

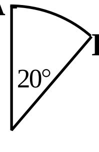
Arc AB is simply a fraction of the circumference.
The fraction depends on the size of the angle at the centre.



e.g.

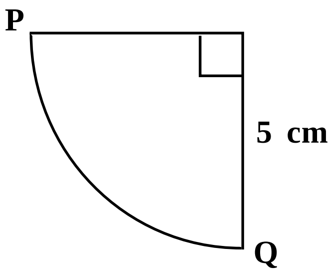


Arc CD has 90° at centre, so ∠COD is $^{90}/_{360}$ of the whole angle at the centre. Therefore, arc CD is $^{90}/_{360}$ of the whole outer circle, the circumference. (or $^{1}/_{4}$)



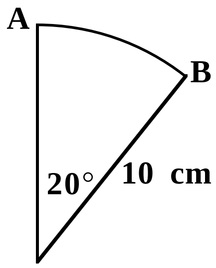
Arc AB has 20° at centre, so ∠AOB is $^{20}/_{360}$ of the whole angle at the centre. Therefore, arc AB is $^{20}/_{360}$ of the whole outer circle, the circumference.

The following 3 examples can be used to illustrate how to find the arc length:



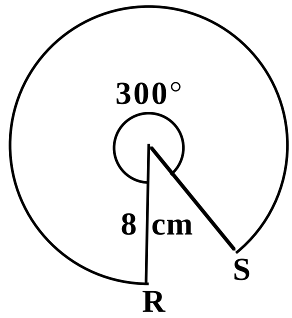
$C = \pi d$
$= 3{\cdot}14 \text{ x } 10$
$= \underline{\underline{31{\cdot}4 \text{ cm}}}$

Arc PQ = $^{90}/_{360}$ x 31·4 cm
= $\underline{\underline{7{\cdot}85 \text{ cm}}}$



$C = \pi d$
$= 3{\cdot}14 \text{ x } 20$
$= \underline{\underline{62{\cdot}8 \text{ cm}}}$

Arc AB = $^{20}/_{360}$ x 62·8 cm
= $\underline{\underline{3{\cdot}49 \text{ cm}}}$

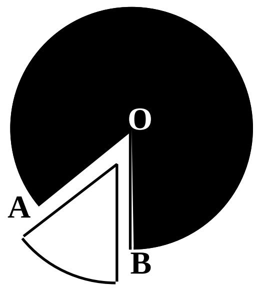


$C = \pi d$
$= 3{\cdot}14 \text{ x } 16$
$= \underline{\underline{50{\cdot}24 \text{ cm}}}$

Arc RS = $^{300}/_{360}$ x 50·24 cm
= $\underline{\underline{41{\cdot}9 \text{ cm}}}$

Exercise 1 may now be attempted. Beware – Q6 for extension only.

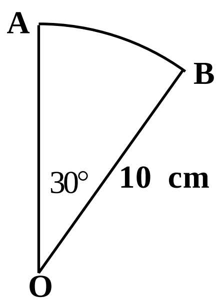
B. The Area of a Sector



Sector AOB is simply a fraction of the whole area of the circle. The fraction depends on the size of the angle at the centre O. A 30° angle at the centre will mean that the sector to which it belongs will have an area of $^{30}/_{360}$ ($^{1}/_{12}$) of the area of the circle.

For example

Calculate the area of sector AOB.



Ans. $A = \pi r^2$ (watch for diameter)
$= 3{\cdot}14 \text{ x } 10 \text{ x } 10$
$= \underline{\underline{314 \text{ cm}^2}}$

Area of Sector AOB = $^{30}/_{360}$ of 314 cm^2
= $\underline{\underline{26{\cdot}2 \text{ cm}^2}}$

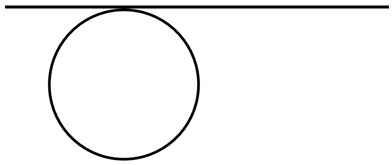
Exercise 2 may now be attempted. Beware – Q8 for extension only.

C. The Relationship between Tangent and Radius

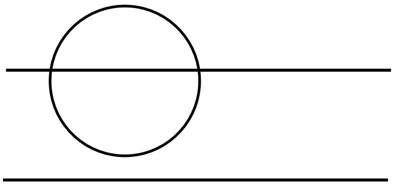
Note: An investigative approach is recommended for this topic.

‘A tangent to a circle is at right angles to the radius through the one point of contact.’

Show this is tangency.



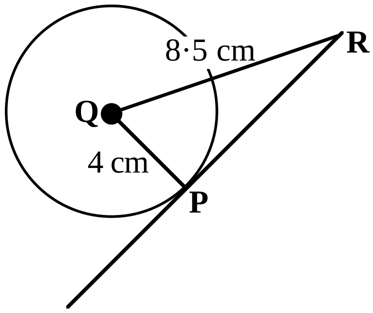
This is not.



An example can then be given to show how the appearance of a right angle may mean the use of ‘Pythagoras’, SOH CAH TOA, angles in a triangle.

Example

PR is a tangent to a circle, centre Q.
Calculate:
(a) length PR
(b) angle PRQ
(c) angle PQR



Ans. the fact that PR is a tangent means that the angle at P is 90°.

(a) By Pythagoras’ Theorem

$$\begin{aligned} PQ^2 &= 8{\cdot}5^2 - 4^2 \\ &= 72{\cdot}25 - 16 \\ &= 56{\cdot}25 \\ PQ &= \underline{\underline{7{\cdot}5\text{ cm}}} \end{aligned}$$

(b) $\sin R = {}^4/_{8\cdot5}$

$\text{angle } R = \underline{\underline{28{\cdot}1°}}$

(c) Angle PQR

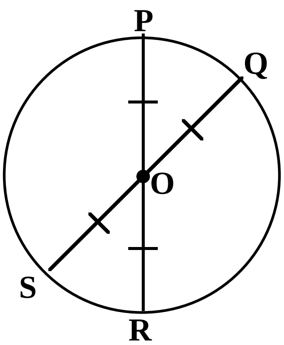
$= 180° - 90° - 28{\cdot}1°$

$= \underline{\underline{61{\cdot}9°}}$

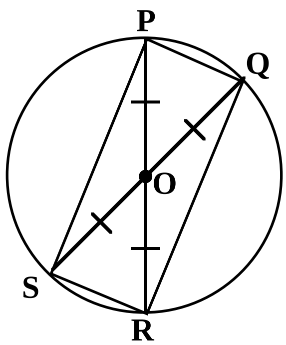
Exercise 3 may now be attempted.

D. Angle in a Semi-Circle

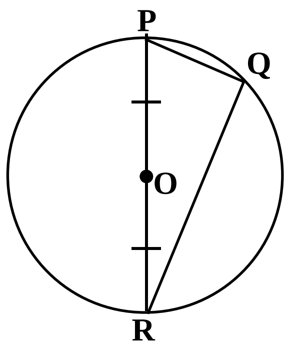
Note: An investigative approach could be used with this topic.



Circle, centre O
diameters PR and QS

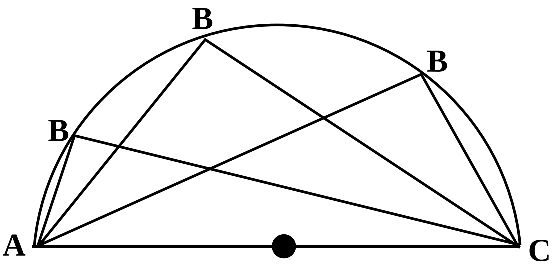


PR and QS are same length
and bisect each other...
PQRS is a rectangle



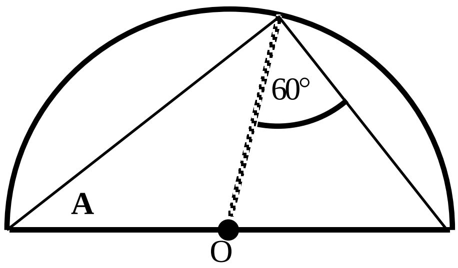
Angle PQR
in a semi-circle
is right angled

By using this picture it should be stressed that no matter where B lies on the circumference, ∠ABC, the angle in the semi-circle is 90°.

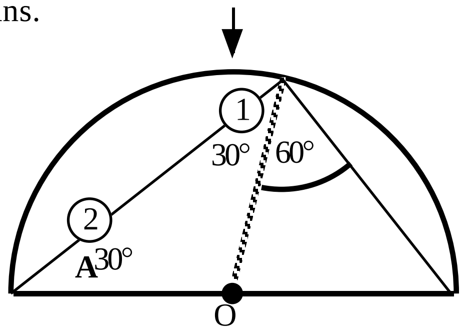


Again though, as with tangency, the appearance of a right angle may mean calculations involving Pythagoras Theorem, SOH CAH TOA etc.

Example 1. Find the value of **A**.



Ans.



Make a sketch
Note the isosceles triangle

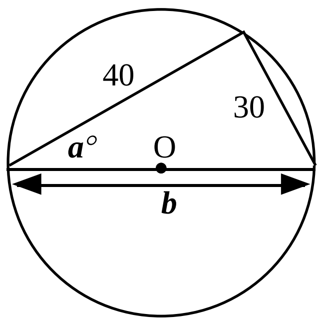
Example 2. Find the values of ***a*** and ***b***.

Ans. $\tan a° = {}^{30}/_{40}$

$a = \underline{\underline{36{\cdot}9°}}$

By Pythagoras' Theorem

$$\begin{aligned} b^2 &= 40^2 + 30^2 \\ &= 1600 + 900 \\ &= 2500 \\ b &= \underline{\underline{50}} \end{aligned}$$



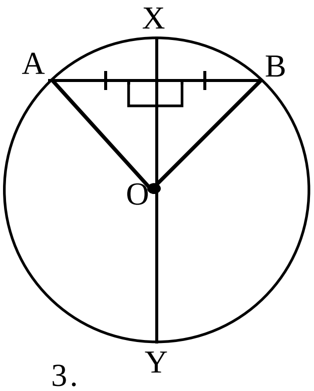
Exercise 4 may now be attempted.

E. The Interdependence of the Centre, Bisector of a Chord and a Perpendicular to a Chord

XY is a diameter of the circle, centre O.

Under reflection in the line XY, B is the image of A.

So, XY bisects AB at right angles.

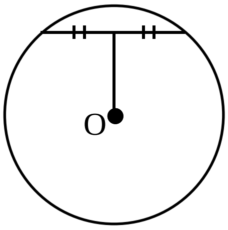


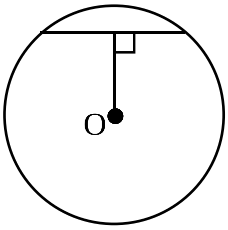
A note can be given of these statements:

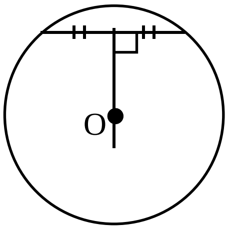
1. Line from centre to mid-point of chord is at right angles to chord

2. Line from centre at right angles to chord bisects the chord

3. Line bisecting chord at right angles goes through the centre.



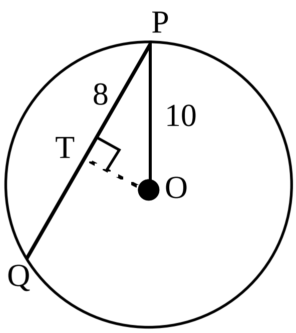




If any one of the statements above is true then the other two are also true.

Example — The radius of a circle is 10 cm.

Calculate the distance from the centre O to the chord PQ which is 16 cm long.



Ans. The dotted line is the required length.

In right angled triangle POT, using Pythagoras' Theorem –

$$
\begin{aligned}
OT^2 &= 10^2 - 8^2 \\
&= 100 - 64 \\
&= 36 \\
OT &= \underline{\underline{6 \text{ cm}}}
\end{aligned}
$$

Note that the question could be rephrased to ask for:

(i) the length of the chord PQ, given length OT and the radius.

(ii) the radius, given chord PQ and length OT.

Exercise 5 may now be attempted.

The Checkup Exercise may now also be attempted.

STUDENT MATERIALS

CONTENTS

CALCULATIONS INVOLVING PERCENTAGES

By the end of this set of exercises, you should be able to

(a) carry out calculations involving percentages in appropriate contexts

(b) round calculations to a required number of significant figures

CALCULATIONS INVOLVING PERCENTAGES

Revision of Basic Percentages

Exercise 1

1. Calculate:

(a) 50% of £25·50	(b) 75% of £28	(c) 25% of £4·40
(d) 10% of £6·80	(e) 20% of £45	(f) 30% of £160
(g) 40% of £18	(h) 60% of £8	(i) 70% of £5
(j) 80% of £9·50	(k) 90% of £2200	(l) 15% of £3
(m) 17·5% of £400	(n) 22·5% of £200	(o) 8·2% of £600
(p) $17^1/_2$% of £20	(q) $8^1/_2$% of £40	(r) $12^1/_2$% of £4

2. What is:

   (a) $33^1/_3$% of £90? (b) $66^2/_3$% of £120?

3. At a dance, only 28% of the 150 people were female.
   How many were: (i) female? (ii) male?

4. A bottle holds 500 millilitres of diluted juice. 96·5% of this is water.
   How many millilitres of water is this?

5. Mavis bought a 750 gram box of chocolates on Saturday afternoon.
   By evening only 15% of them were left.
   What weight of chocolates remained?

6. The village of Elderslie has 3800 residents. Only 2% of them attended a local meeting.

   (a) How many villagers attended the meeting?
   (b) How many did not bother to go?

7. A jet was flying at 32 000 feet when one of its engines failed.
   The jet dropped by 42% in height. By how many feet did it drop?

8. When David was 14 he was 140 cm tall. Over the next year he grew by 2·5%.
   What was his height when he reached 15 years?

9. At Stanford City Football Club, 95% of its home support are season ticket holders.
   The stadium has room for 44 200 home supporters.
   How many home supporters do not have a season ticket?

10. Mrs. Nicolson borrows £1200. She must pay back the loan plus interest at a rate of 9% per year.

Calculate the amount she has to pay if she wishes to pay back the loan (plus interest) in:

(a) 1 year (b) 6 months (c) 9 months (d) 4 months (e) 5 months.

11. Of the 40 guests at a party, 32 of them were women. What percentage were women?

12. Of the 180 cars which took part in a rally, 45 of them were green.

What percentage of them were not green?

13. From my weekly pay of £280, I spend £84 in rent.

What percentage of my pay do I spend on rent?

14. 2000 people were stuck at the airport, due to flight delays. The first flight to leave was to Orkney. It left carrying 72 of the people.

What percentage of the people already at the airport remained there?

A. Compound Interest

Exercise 2

1. The following people have opened up Investment Accounts and are leaving their money to grow with compound interest.

For each, calculate the total amount in their account after the stated period.

(a) Anna, deposits £1200 for 3 years at a rate of interest of 5% per annum.

(b) Judy, deposits £650 for 2 years at a rate of interest of 4% per annum.

(a) Anna, deposits £50 for 2 years at a rate of interest of 2% per annum.

2. Calculate the total compound interest earned on a deposit of £450 for 3 years at 4% p.a.

(The interest should only be calculated on complete pounds of principal).

3. Conrad James deposited £500 in his bank and left it there for 3 years, gaining interest each year. Unfortunately, the interest rate dropped each year – from 10% in the first year to 8% in the second year to 5% in the third year.

When he withdrew all his money at the end of year three how much did he receive?

4. A businessman borrowed £8000 at a rate of interest of 5% per annum. He made payments at the end of each year based on the sum outstanding at the end of that year.

At the end of the first year and again at the end of the second year he paid back £3000.

How much had he to pay at the end of the third year to clear the debt ?

5. Mary Telfer deposited £250 in her bank and left it there for 3 years, gaining interest each year. The interest rate rose from 4% in the first year to 5% in the second year, but fell drastically to 1% in the third year.

   She took out all her money atthe end of year 3.

   How much did she withdraw ?

6. Mrs. Donaldson deposits £750 in a Building Society which pays 3% compound interest <u>half yearly</u>.

   Mrs. Edgar, her neighbour, puts her £750 into another Building Society where her investment gains 6% compound interest annually.

   (a) How much will each have in their Building Society after 1 year?

   (b) Is a rate of 3% compound interest paid half yearly equivalent to a rate of 6% compound interest paid annually? Explain!

7. Use the $\boxed{y^x}$ key on your calculator for this question.

   Calculate the compound interest on £3340 for 10 years at 6·5% per annum.

8. How many years would it take for £50 to (at least) double at a rate of 10% compound interest?

B. Appreciation and Depreciation

Exercise 3

1. Mr. and Mrs. Pollard bought a semi-detached house for £60 000.
   In each of the following two years its value appreciated by 10%.

   How much was the house worth after the two years?

2. Newly weds Jack and Jane Jones bought a flat for £55 000. It appreciated in value by 7·5% p.a. for the next two years until they sold it.
   How much did they get for their flat? (to the nearest £)

3. The Herald’s bought a bungalow for £110 000.

   It appreciated in value for the next three years by 8% in year 1, by 6·5% in year 2 and by 5% in year 3.

   How much was the bungalow worth after three years?
   (to the nearest £).

4. Miss Hamilton retired to a villa which she bought for £68 500.

   The value of the villa rose by 5·4% each year.

   How much was the villa worth after 2 years? (to the nearest £)

5. Bert, the garage owner, bought a second-hand breakdown truck for £5000.

The truck lost 40% of its value during the first year, 20% during the second year and 10% during the third year.

How much was the breakdown truck worth after these 3 years?

6. A contractor bought a digger for £75 000. It depreciated by 75% in year one, by 40% in year two and by 20% in year three.

What was the digger worth after 3 years?

7. The value of a photocopier in a school office depreciates by 42% annually.

How much will an £18 000 copier be worth at the end of two years?

8. A small conservatory was valued at £8 000 in 1997 and again a year later at £8 336.

Calculate how much it had increased in value, and express this as a percentage of its 1997 value.

9. Mr. Able owns a detached villa in Melrose.

In 1996 he had the house valued - £85 000.

By 1997 it had depreciated by 15%, and by 1998 it was worth 20% more than in 1997.

Calculate:

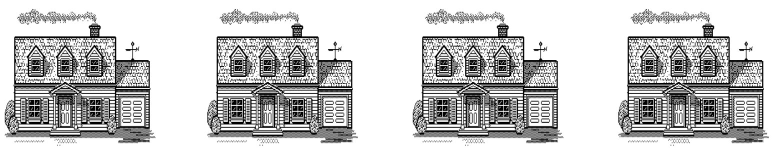
(a) its value in 1998.

(b) the percentage change in value from 1996 to 1998.

10. Calculate the percentage appreciation of the value of this detached villa:

(a) from 1996 to 1997.

(b) from 1996 to 1999.



1996	**1997**	**1998**	**1999**
£120 000	**£126 000**	**£128 520**	**£129 600**

11. Calculate the percentage depreciation of the value of this car:
   (a) from 1995 to 1996.
   (b) from 1997 to 1998.
   (c) from 1995 to 1999.

1995	**1996**	**1997**	**1998**	**1999**
£12 000	**£4 800**	**£2 400**	**£1 920**	**£1 800**

12. The value of an antique jug rose by 5% to £10 500.
    Work out its previous value. (<u>not</u> £9 975!)

C. Significant Figures

Exercise 4

1. Round the following numbers to one significant figure (1 sig. fig.).

(a) 4269	(b) 14 774	(c) 17	(d) 487
(e) 18 152	(f) 2085	(g) 7510	(h) 6551
(i) 42 670	(j) 451	(k) 14 308	(l) 24 859
(m) 6 890 000	(n) 55 847 155	(o) 38 749 886 541	(p) 25

2. Round the following numbers to two significant figures (2 sig. figs.).

(a) 5187	(b) 24 885	(c) 221	(d) 555
(e) 19 352	(f) 2065	(g) 7650	(h) 6549
(i) 42 501	(j) 448	(k) 78 209	(l) 29 899
(m) 6 890 000	(n) 55 847 155	(o) 38 749 886 541	(p) 351

3. Round the following numbers to three significant figures (3 sig. figs.).

(a) 8181	(b) 24882	(c) 2217	(d) 5554
(e) 19 551	(f) 2077	(g) 7682	(h) 6149
(i) 42 552	(j) 4499	(k) 78 209	(l) 29 897
(m) 6 893 000	(n) 55 847 155	(o) 38 749 886 541	(p) 35 150 001

4. Round <u>each</u> of the following decimals to:
   (i) 1 significant figure
   (ii) 2 significant figures
   (iii) 3 significant figures

(a) 8·33333	(b) 23·81558	(c) 1·53097	(d) 347·502

Exercise 5

In this exercise, round the answers to the required number of significant figures.

1. For each person, calculate the total amount in their account after the stated period.
   (a) Janice deposits £2000 for 3 years in her Investment Account at a compound interest rate of 5% per annum. (2 sig figs.)
   (b) Rob deposits £1500 for 2 years in his Investment Account at a compound interest rate of 4% per annum. (1 sig fig.)
   (c) Quasim deposits £3000 for 4 years in his Investment Account at a compound interest rate of 10% per annum. (3 sig figs.)

2. Sally James deposited £800 in her bank and left it there for 3 years, gaining interest each year. The interest rate was 10% in the first year, 5% in the second year and 3% in the third year.

   When she withdrew all her money at the end of year 3 how much did she receive? (answer to 2 sig figs.)

3. Calculate the compound interest on £6580 for 15 years at 3% per annum.

   Use the $\boxed{y^x}$ key on your calculator. (3 sig figs.)

4. Mr. and Mrs. Greig bought a detached house for £85 000.

   In each of the following two years its value appreciated by 8·5%.

   How much was the house worth after the two years? (2 sig fig.)

5. The Thomson's bought a seaside apartment for £32 500.

   It appreciated in value for the next three years by 10% in year one, by 4% in year two and by 3% in year three.

   How much was the apartment worth after three years? (2 sig figs.)

6. Ami bought a small aircraft with the money left to her by an old aunt. She paid £104 000.

   The plane lost 50% of its value during the first year, 35% during the second year, 20% during the third year and 12·5% during the fourth year.

   How much was the aircraft worth after these 4 years? (3 sig figs.)

cont'd ...

7. This table shows the value of a dishwasher, bought new in 1995, over a four year period.

Year	1995	1996	1997	1998	1999
Value	£600	£320	£240	£140	£50

Calculate the percentage depreciation of the value of the dishwasher:

(a) from 1995 to 1996. (2 sig figs.)
(b) from 1997 to 1998. (3 sig figs.)
(c) from 1995 to 1999. (1 sig fig.)

8. Calculate the percentage appreciation of the value of this precious teddy:
(a) from 1996 – 1997. (1 sig fig.)
(b) from 1997 – 1998. (2 sig figs.)
(c) from 1996 – 1999. (1 sig fig.)

1996
£500

1997
£542

1998
£700

1999
£978

Checkup for Calculations Involving Percentages

1. Calculate the total compound interest earned on a deposit of £200 for two years when the annual interest rate was 8%.

2. Frank Graham deposited £6000 in his bank and left it there for 3 years, gaining interest each year.
   The interest rate fell from 7% in the first year to 5% in the second year, but rose to 10% in the third year.

   He withdrew all his money at the end of year 3.

   How much did he then receive? Give your answer correct to two significant figures.

3. A company director borrowed £20 000 and was charged a rate of interest of 3% per annum, calculated on the sum outstanding at the beginning of the year.

   At the end of the first year and again at the end of the second year he paid back £10 000.

   How much had he to pay at the end of the third year to clear the debt?

   Give your answer correct to three significant figures.

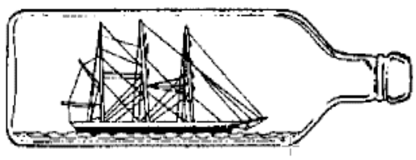
4. Calculate the compound interest on £200 for 25 years at 5% per annum.

   Give your answer correct to one significant figure.

5. Julie Rocks bought a flat in Peterhead for £20 000. It increased in value over the next three years at an annual rate of 6%.

   What was the value of the flat at the end of these 3 years?

   Give your answer correct to three significant figures.

6. 

   This antique ship in a bottle appreciated in value over a four year period by consecutive rates of 10%, 20%, 50% and 100% per annum.

   What was it worth after 4 years if its original price was £100.

7. A yacht was purchased new, at a cost of £250 000.

   It fell by 15% of its value each year over the next three years and at the end of the fourth year it was found to be worth £100 000.

   (a) By how much money did the yacht depreciate during the fourth year?

   (b) Calculate the percentage depreciation over the first three years, giving your answer correct to two significant figures.

8. Mrs. Penny Black owns a treasured stamp which was valued, 40 years ago, at £300.
   It is estimated that the stamp has grown in value by at least 10% per annum since then.

   What is the estimated value of the stamp today?

   Give your answer correct to three significant figures.

VOLUMES OF SOLIDS

By the end of this set of exercises, you should be able to

(a) calculate the volumes of a prism, cone and sphere

VOLUMES OF SOLIDS

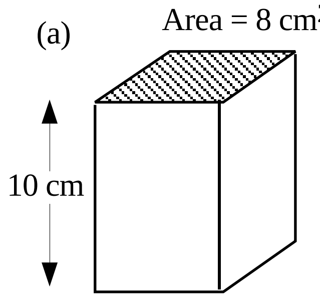
$$\text{Volume}_{\text{prism}} = \text{Area}_{\text{base}} \times \text{height}$$

A. Volume of a Prism

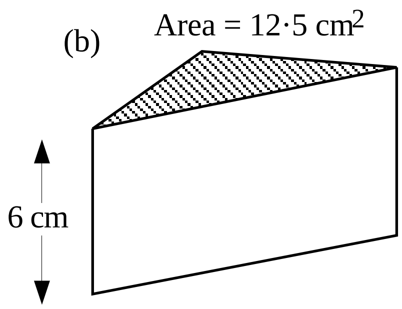
Exercise 1

1. For each of the following prisms, the area of the base or end face is given. Calculate the volumes of the prisms:

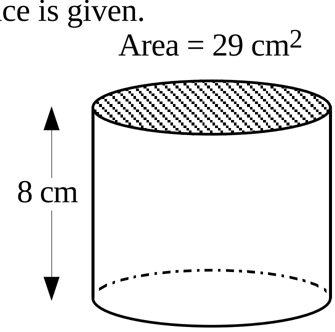
(a)



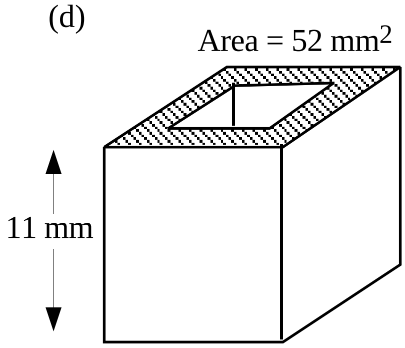
(b)



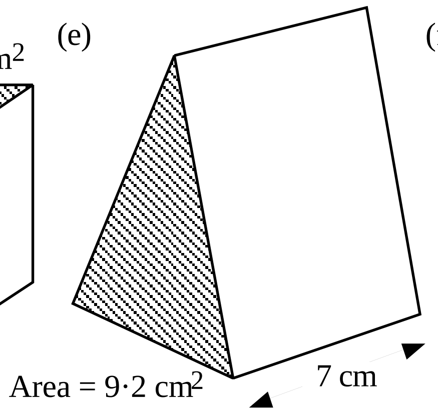
(c)



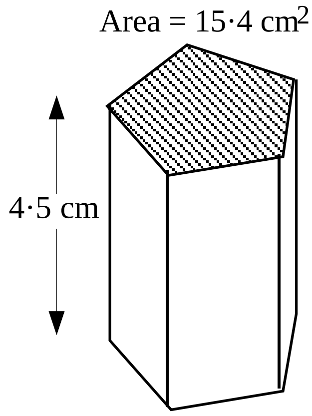
(d)



(e)

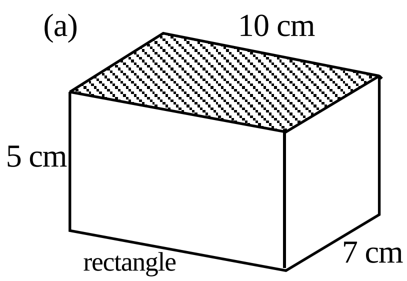


(f)

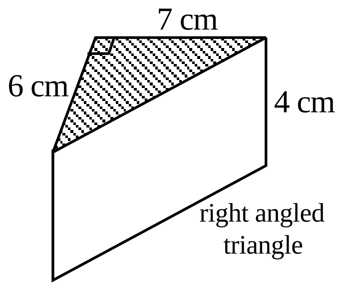


2. This time you must calculate the shaded area first, then find the volumes of the prisms.

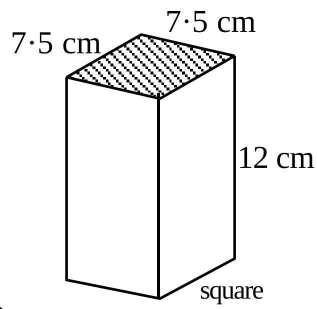
(a)



(b)



(c)



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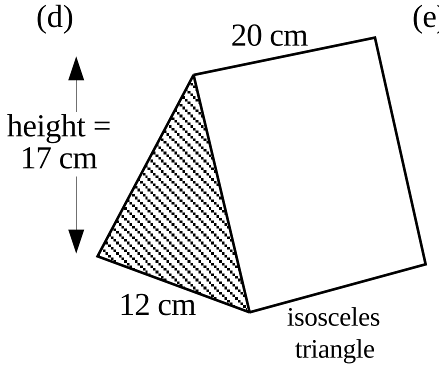

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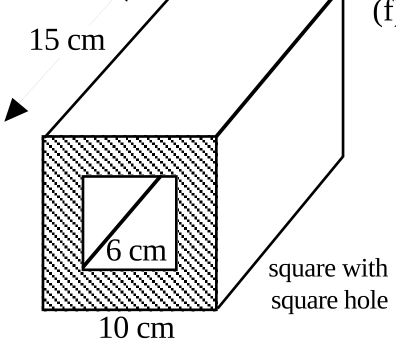
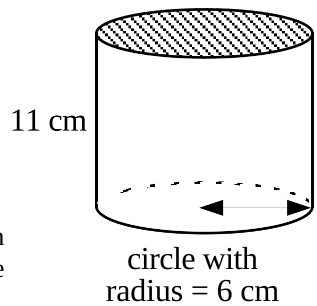


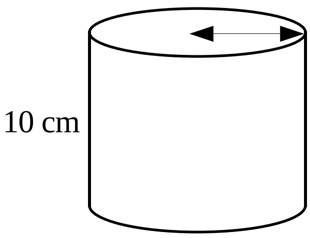
(f)



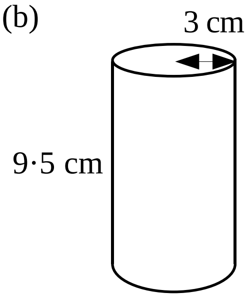
3. The **cylinder** – a special prism.
Calculate the volumes of the following cylinders:

$\boxed{\text{Volume}_{\text{(cylinder)}} = \pi r^2 h}$

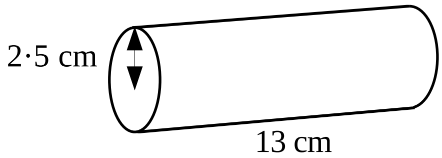
(a)



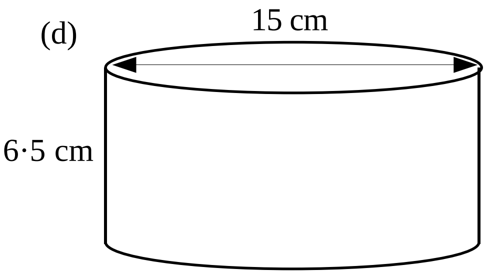
(b)



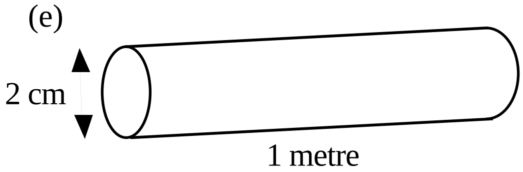
(c)



(d)



(e)

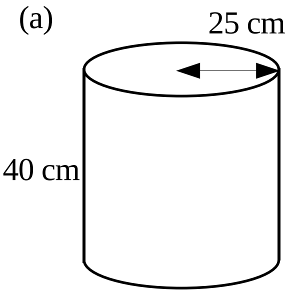


4. **Remember:**

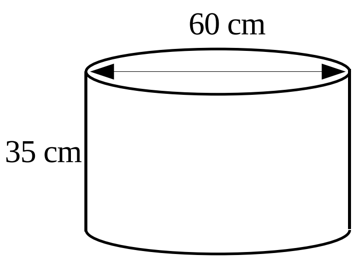
$\boxed{1 \text{ cm}^3 = 1 \text{ ml}; \quad 1000 \text{ cm}^3 = 1000 \text{ ml} = 1 \text{ litre}}$

How many litres of water will the following drums hold?

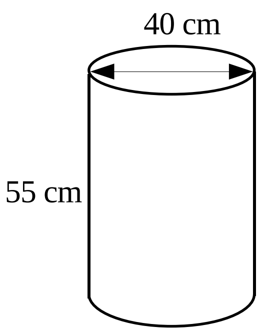
(a)



(b)

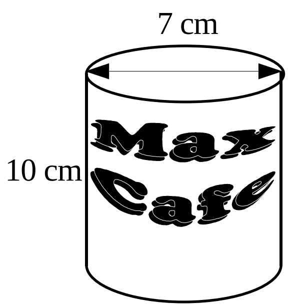


(c)

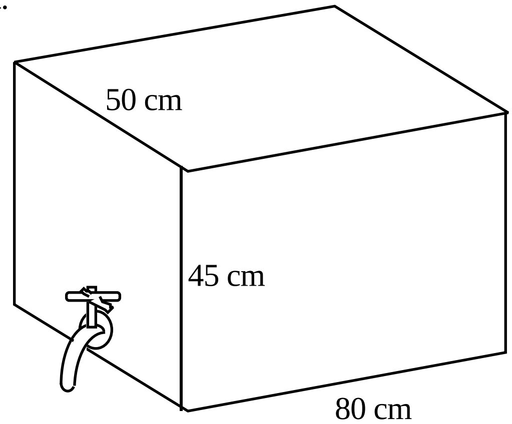


5. A cylindrical tin of Maxcafe Coffee is 10 centimetres high and has a base diameter of 7 centimetres.

What is the volume of coffee in the tin when it is full?

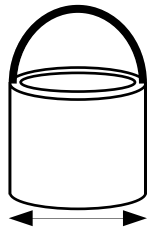


6. This rectangular storage tank is full of white paint.



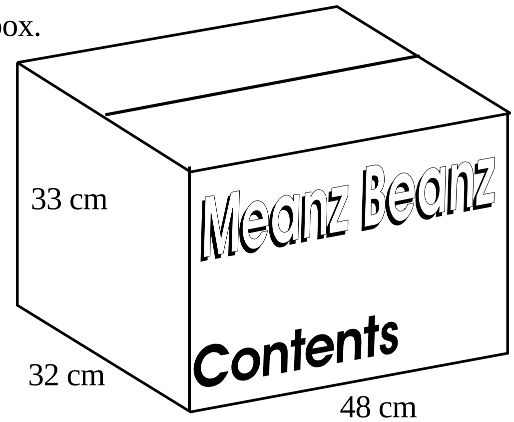
(a) Calculate the volume of paint in the tank in cubic centimetres (cm^3).

(b) Calculate the volume of this cylindrical paint tin.

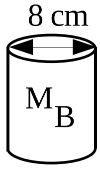


(c) How many times can the paint tin be <u>completely</u> filled from the tank?

7. Meanz Beanz tins are packed into this cardboard box.



(a) How many tins can be placed on the bottom layer?

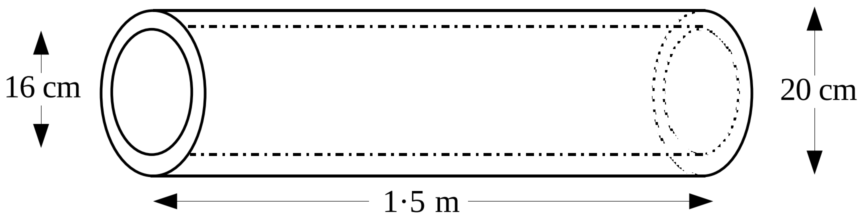


(b) How many layers will there be?

(c) How many tins can be packed in the box altogether?

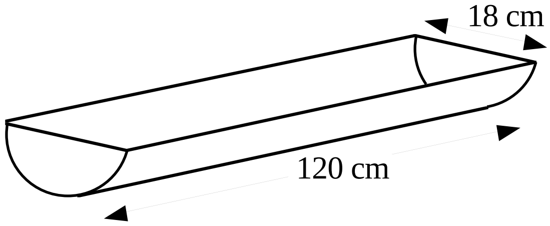
(d) How much air space in the box is there around all the tins?

8. This cast iron pipe has an internal diameter of 16 centimetres and an outside diameter of 20 centimetres. The pipe is 1·5 metres long.



Calculate the volume of iron needed to make the pipe.

9. How much liquid feeding will this semi-cylindrical pig-trough hold?



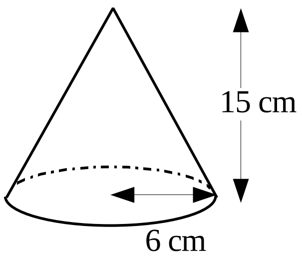
B. Volume of a Cone

$$\text{Volume}_{\text{(cone)}} = \tfrac{1}{3}\pi r^2 h$$

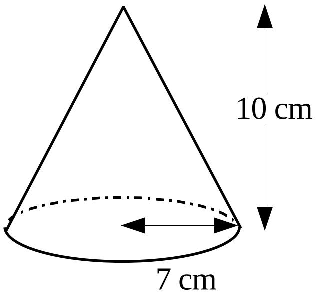
Exercise 2

1. Calculate the volumes of the following conical shapes:

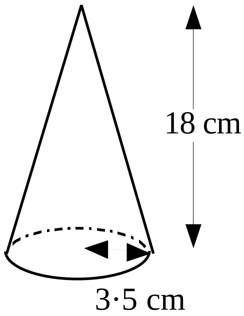
(a)



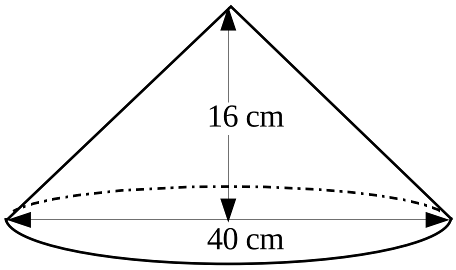
(b)



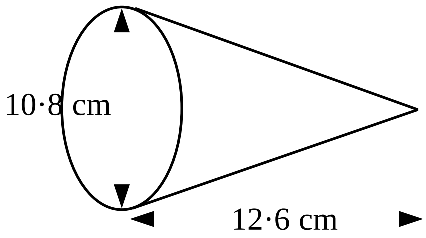
(c)



(d)



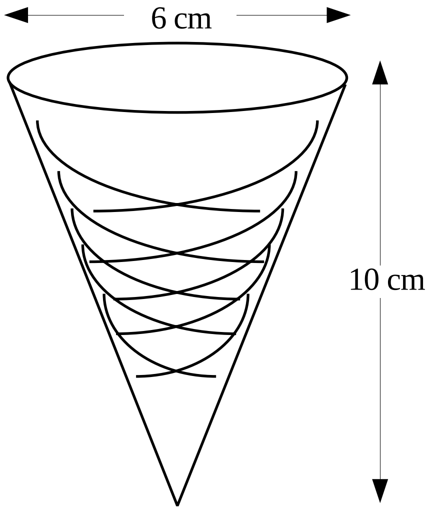
(e)



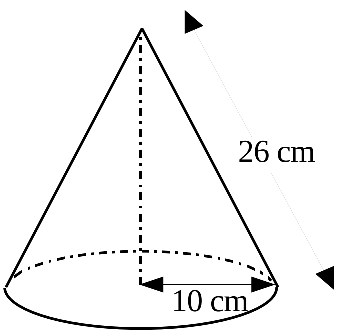
2. The wafer of an ice-cream cone has a diameter of 6 centimetres.

   The cone is 10 centimetres high.

   Calculate the volume of the cone.



3.



The ‘sloping’ height of this cone is 26 cm.

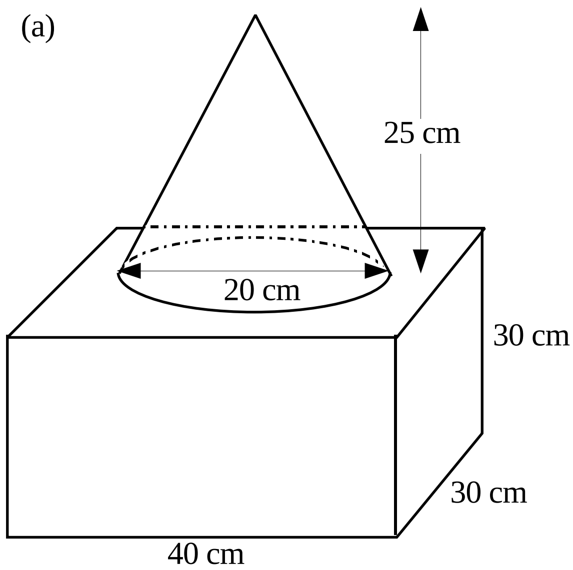
The base radius is 10 cm.

(a) Calculate the height of the cone.

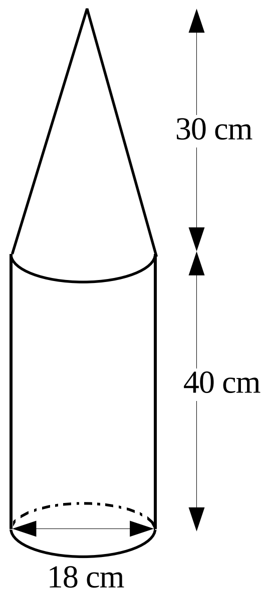
(b) Calculate the volume of the cone.

4. Calculate the total volumes of the following shapes.

(a)



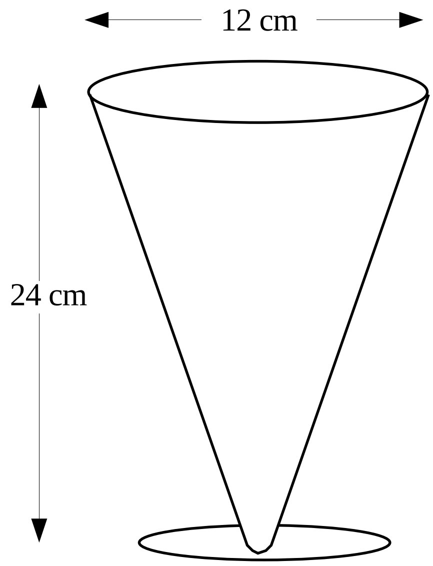
(b)



5. Water is poured into this conical flask at the rate of 50 millilitres per second.

(a) Calculate the volume of the flask.

(b) How long will it take, to the nearest second, to fill the flask to the top?

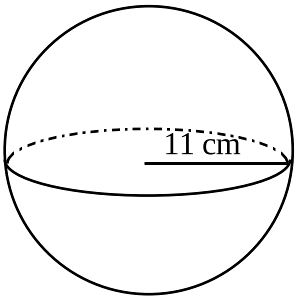


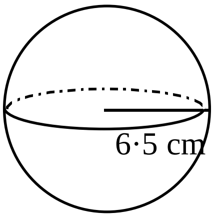
C. Volume of a Sphere

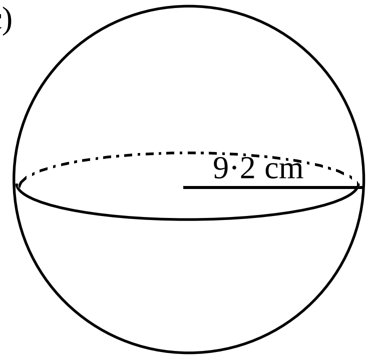
Volume $_{(sphere)}$ = $^4/_3\pi r^3$

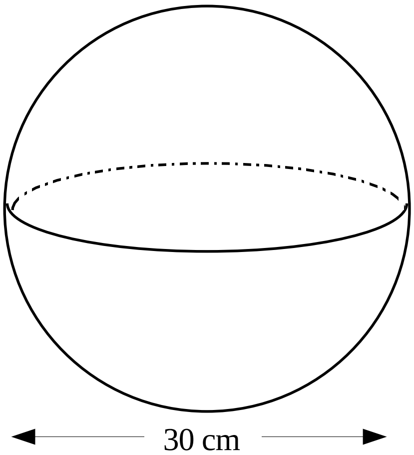
Exercise 3

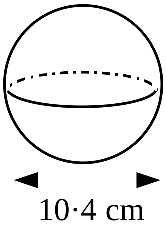
1. Calculate the volumes of the following spheres:

(a) 

(b) 

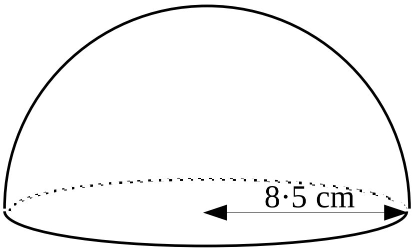
(c) 

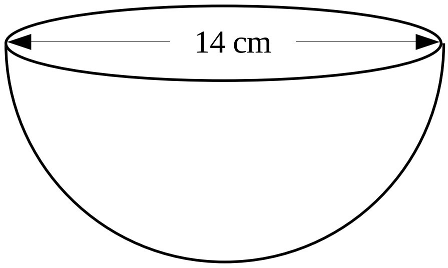
(d) 

(e) 

2. This football is fully inflated. Calculate the volume of air inside the football.

3. Calculate the volumes of these two 'hemispheres':

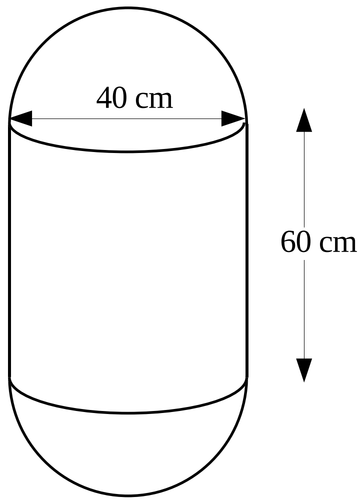
(a) 

(b) 

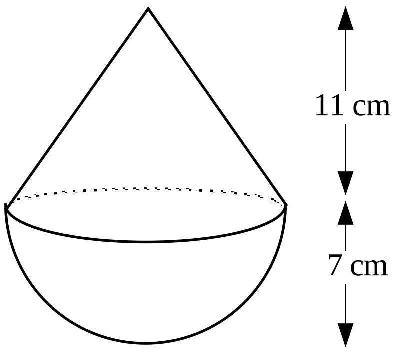
4. (a) Calculate the volume of water which can be stored in this copper hot water tank in cm^3.

The tank consists of a cylinder with two hemispherical ends.

(b) How many litres of water will it hold? ($1cm^3$ = 1 ml; 1000 ml = 1 litre).



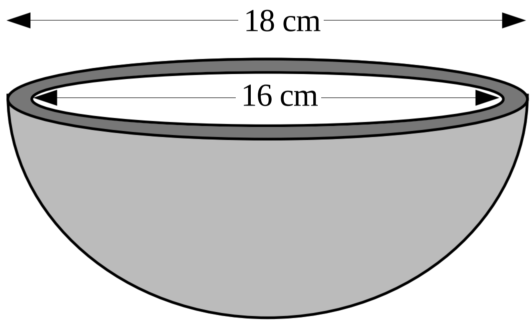
5.



Calculate the volume of this child's rocking toy which consists of a cone on top of a hemisphere.

6. This decorative wooden fruit bowl is in the shape of a hollowed out hemisphere.

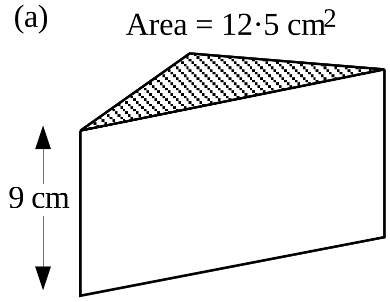
Calculate the volume of wood required to make it.



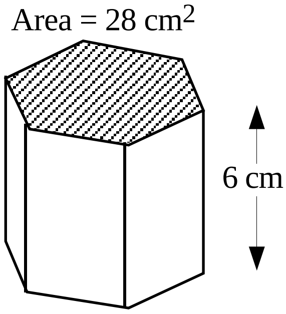
Checkup for Volumes of Solids

1. Calculate the volumes of the following prisms:

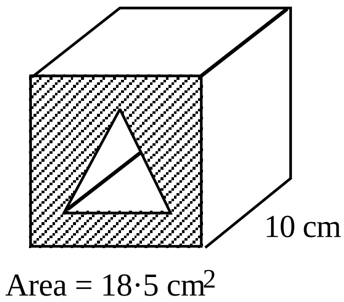
(a)



(b)

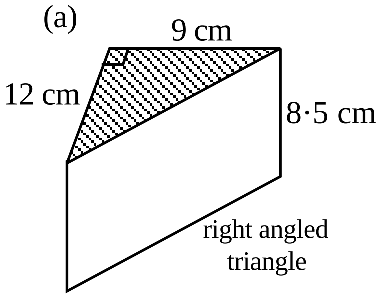


(c)

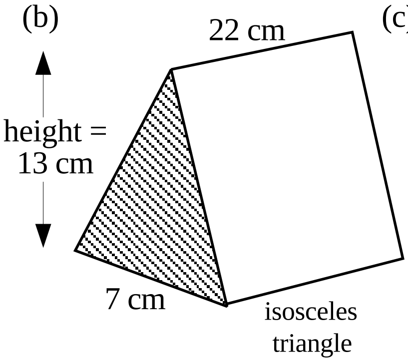


2. Calculate the shaded areas and use them to find the volume of each shape.

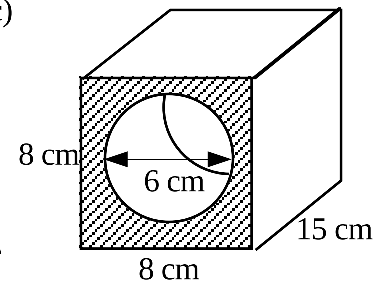
(a)



(b)



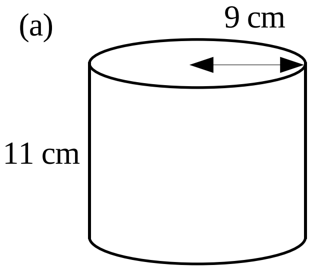
(c)



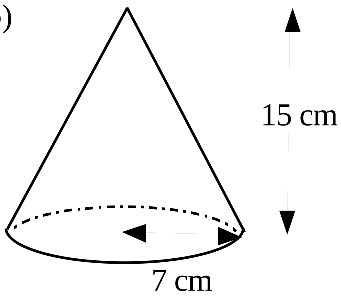
3. Calculate the volumes of the following shapes:

Vol (cylinder) = $\pi r^2 h$
Vol (cone) = $\frac{1}{3}\pi r^2 h$
Vol (sphere) = $\frac{4}{3}\pi r^3$

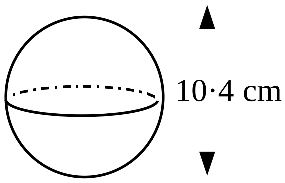
(a)



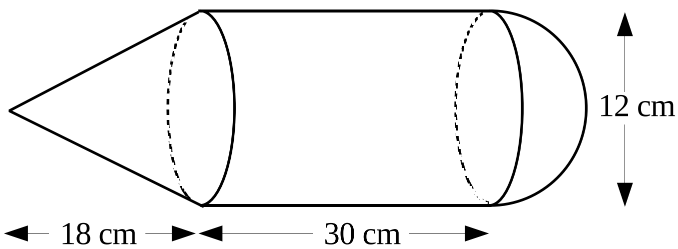
(b)



(c)



4. This shape consists of a cone, a cylinder and a hemisphere. Calculate its total volume.



LINEAR RELATIONSHIPS

By the end of this set of exercises, you should be able to

(a) determine the gradient of a straight line

(b) sketch a straight line given its equation in the form $y = \mathrm{a}x + \mathrm{b}$

(c) determine the equation of a straight line in the form $y = \mathrm{a}x + \mathrm{b}$ from its graph

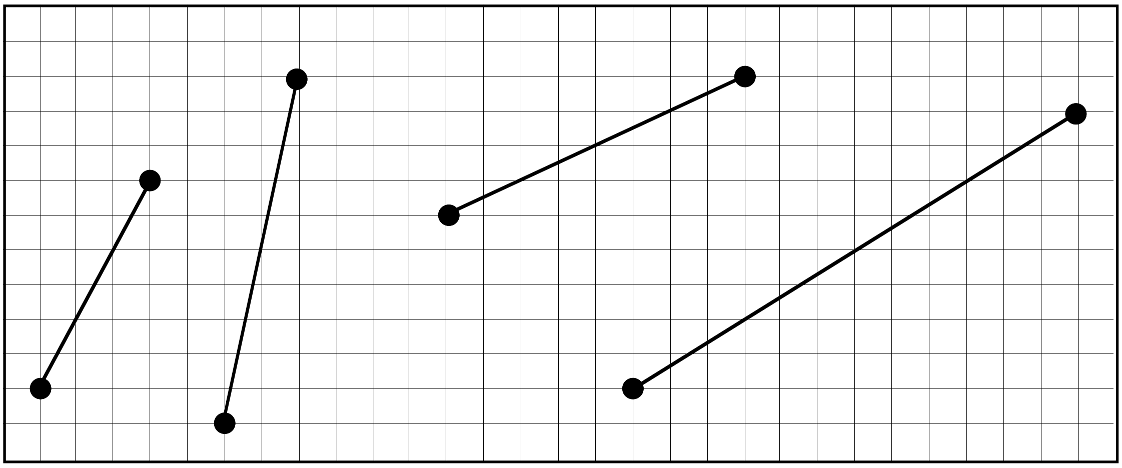
LINEAR RELATIONSHIPS

A. The Gradient of a Line

Exercise 1

$$\text{gradient} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

1. Find the gradient of each line using the formula:



2. For each of the following pairs of points:
   (i) draw a (small) coordinate diagram,
   (ii) plot the two points and join them to form a straight line,
   (iii) calculate the gradient of the line joining the two points.

   (a) P(1,1), Q(3,9) (b) A(3,0), B(5,6)
   (c) R(–3,1), S(5,5) (d) L(–4,–1), M(2,3)

3. Calculate the gradients of the lines joining the following pairs of points:

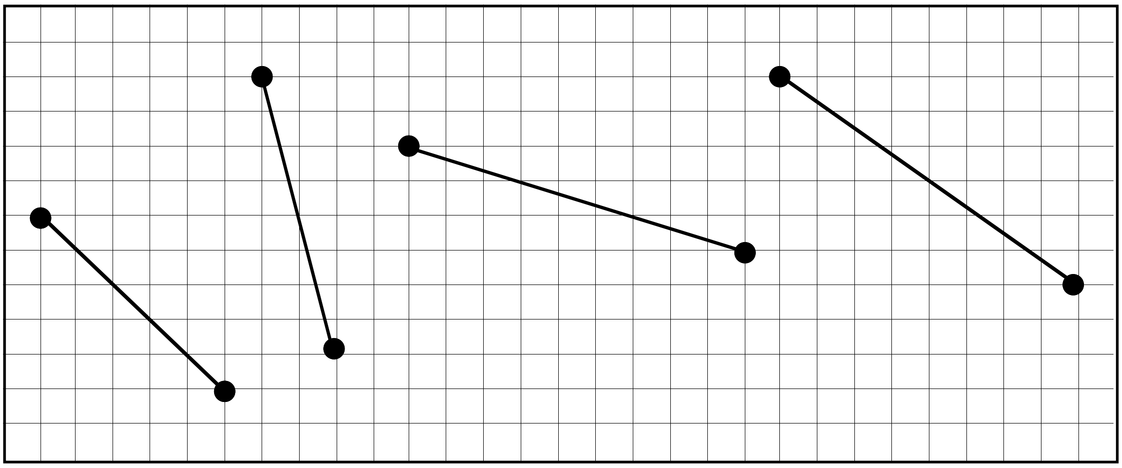
   (a) C(1,5), D(7,7) (b) U(0,3), V(12,7)
   (c) J(–1,–6), K(1,6) (d) O(0,0), T(5,15)

   So far, all the lines you have met in this exercise have had gradients which were positive.

4. Describe how a line with a negative gradient differs in shape from that of a line with a positive gradient.

cont'd .....

5. Calculate the gradient of each line.



6. Calculate the gradients of the lines joining the following points.
(Some are positive, some negative).

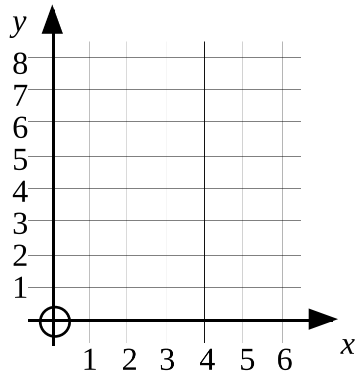
(a) A(1,6), B(6,1)	(b) D(0,7), E(2,3)	(c) G(–2,5), H(1,–4)
(d) J(–6,–3), K(3,0)	(e) M(–6,0), N(0,–4)	(f) P(1,–1), Q(3,1)
(g) S(–1,10), T(3,–2)	(h) V(–6,–10), W(2,–6)	(i) Y(–12, 5), Z(3,0)

7. (a) On a small coordinate diagram plot the two points A(1,3) and B(6,3).
(b) Find the gradient of the line joining A and B using your formula.
(c) Comment on the connection between the shape (slope) of the line drawn in part (a) and the corresponding value of its gradient as calculated in part (b).

B. Sketching Lines in the form $y = ax + b$

Exercise 2

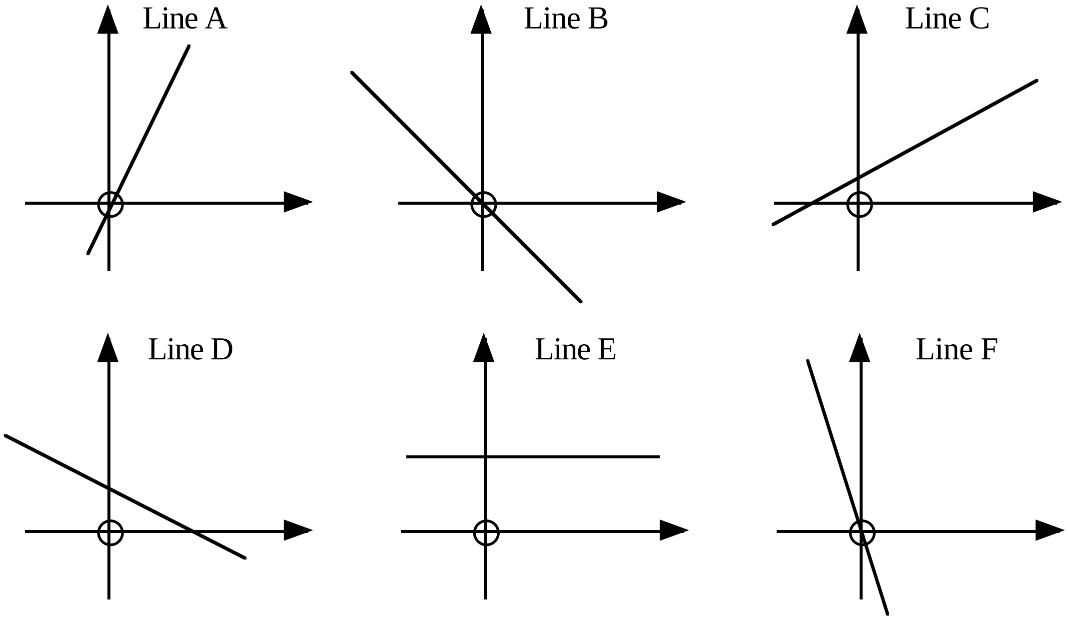
1. Drawing the line $y = 2x + 1$:
   (a) Make a copy of this coordinate diagram.
   (b) Where does the line $y = 2x + 1$ cut the y - axis? (plot this point).
   (c) The gradient of the line is **<u>2</u>**. From your first plotted point, move **1** box right and **<u>2</u>** boxes up. Plot this 2nd point.
   (d) Join your 2 points and extend the line.
   (e) Label the line $y = 2x + 1$.



2. Draw the following lines, labelling each one carefully.

(a)	$y = 3x + 2$	(b)	$y = 4x - 3$
(c)	$y = x + 5$	(d)	$y = {}^{1}/_{2}x + 4$
(e)	$y = -2x + 1$	(f)	$y = -3x - 5$
(g)	$y = -x + 3$	(h)	$y = {}^{3}/_{4}x + 1$

3. Look at the 6 lines shown and the list of 6 gradients given below

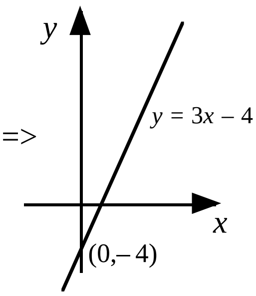


Gradients: $a_1, = {}^{1}/_{2}$, $a_2 = -3$, $a_3 = -{}^{1}/_{2}$, $a_4 = 0$, $a_5 = -1$, $a_6 = 2$

Match up the lines (A, B, C, D, E, F) with the gradients (a_1, a_2, a_3, a_4, a_5, a_6).

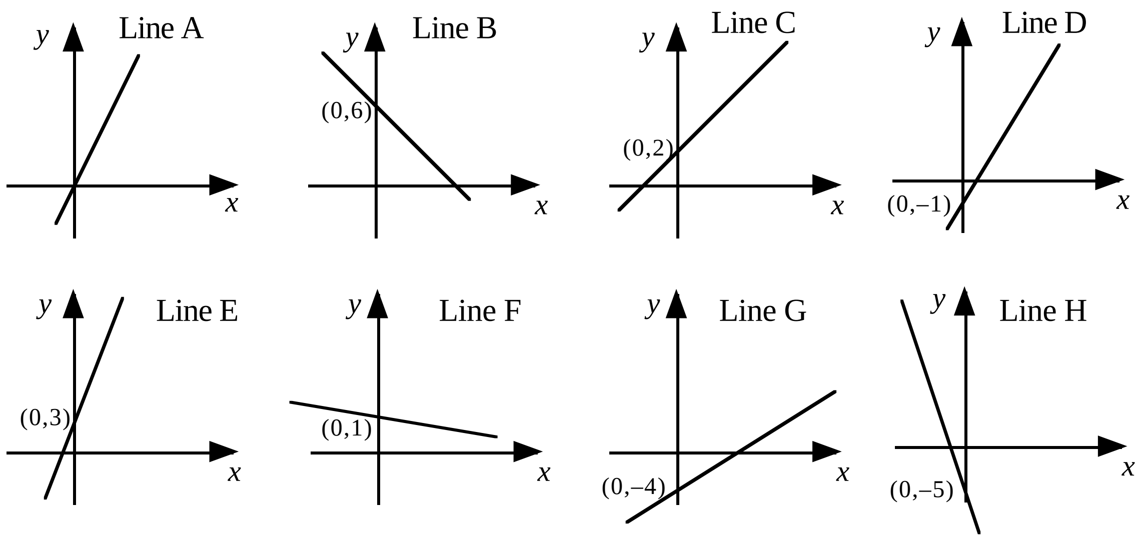
4. This time, simply make a neat <u>sketch</u> of the given line, indicating where it cuts the y - axis.

example: $y = 3x - 4$ =>



(a)	$y = x + 3$	(b)	$y = 2x - 3$
(c)	$y = {}^{1}/_{2}x + 6$	(d)	$y = -2x + 3$
(e)	$y = -x - 4$	(f)	$y = 6x - 6$
(g)	$y = {}^{1}/_{5}x + 2$	(h)	$y = -{}^{1}/_{2}x + 4$
(i)	$y = -4x - 3$	(j)	$y = {}^{4}/_{3}x - 1$

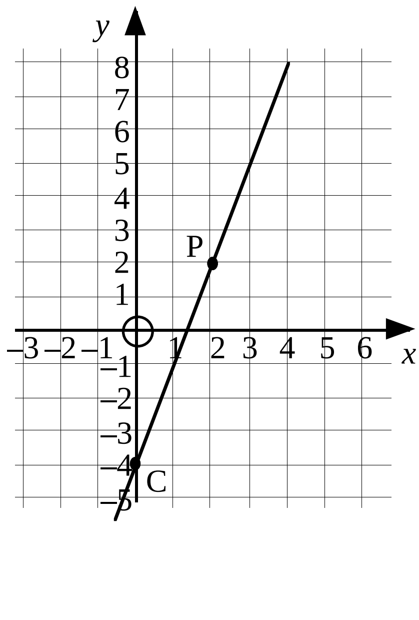
5. Look at the following sketches of 8 lines and the list of 8 equations.
Match each line to its corresponding equation.



Lines: $y = 2x - 1$, $y = 5x + 3$, $y = 3x$, $y = {}^{1}/_{2}x - 4$,
$y = -2x - 5$, $y = -x + 6$, $y = -{}^{1}/_{4}x + 1$, $y = x + 2$.

C. Determining the equation of a line in the form *y* = a*x* + b

Exercise 3



1. Determine the equation of the line shown opposite

   Step 1 Start always with the general equation of any line:

   => $y = ax + b$

   Step 2 Pick out the coordinates of where the line cuts the *y* - axis – (0,...)

   Use this to begin to write down the line's equation:

   => $y = ax - ...$

   Step 3 Find the gradient of the line by using <u>any</u> two points on the line e.g. C and P.

   Use this to complete your equation:

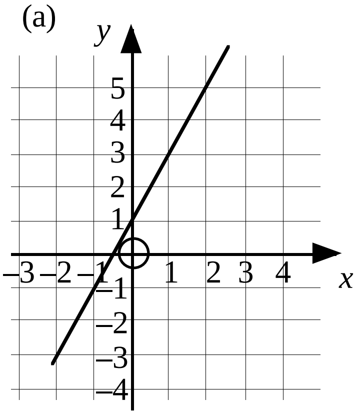
   => $y = ..x - ...$

2. On the next page there are drawings of six lines.
   Use the technique shown in question 1 to determine their nature.

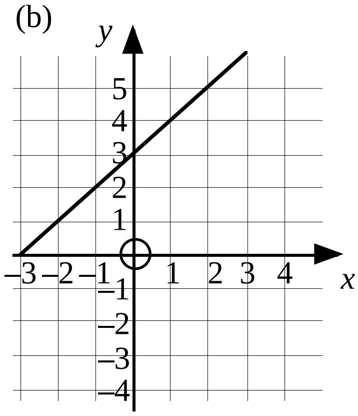
cont'd ...

2. cont'd...

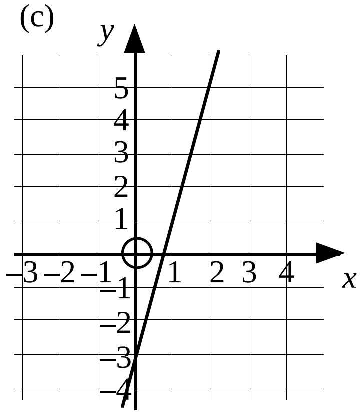
(a)



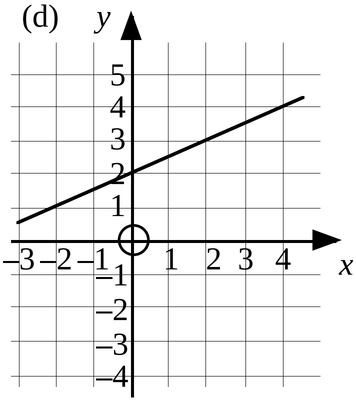
(b)



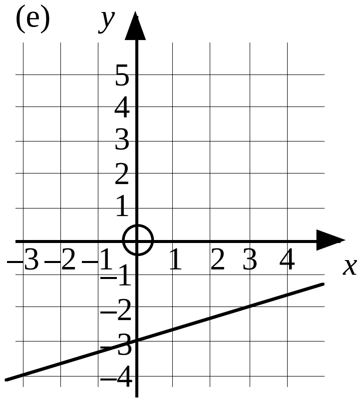
(c)



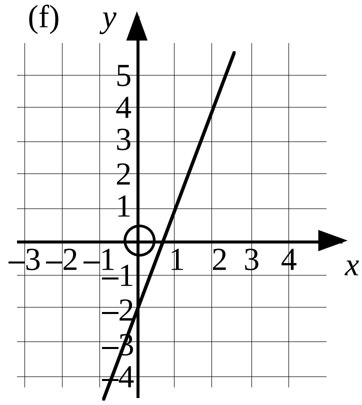
(d)



(e)

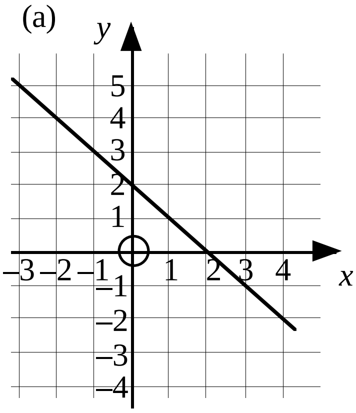


(f)

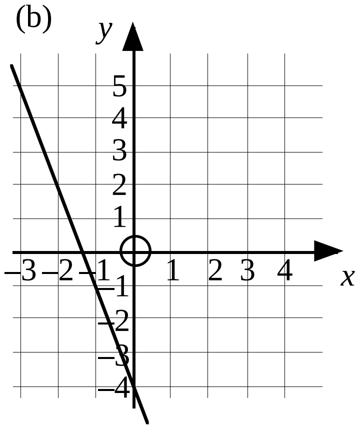


3. The following lines all have negative gradients. Use the same technique as shown in question 1 to determine their equations.

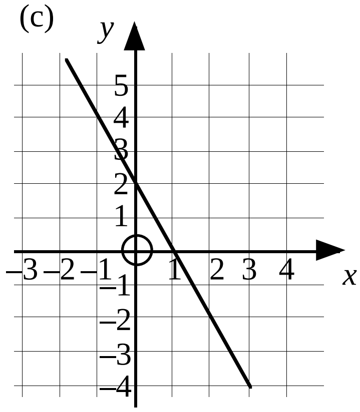
(a)



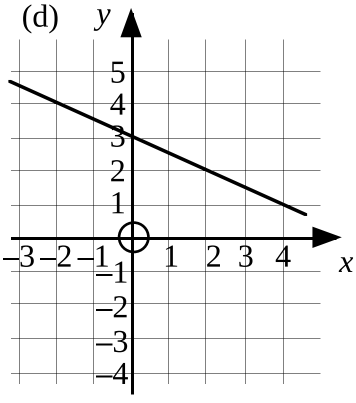
(b)



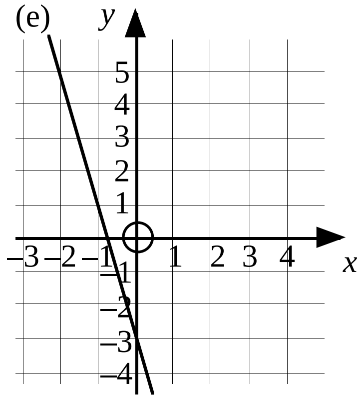
(c)



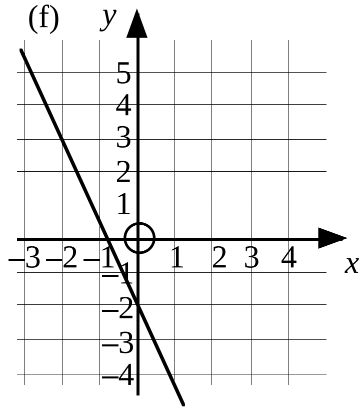
(d)



(e)



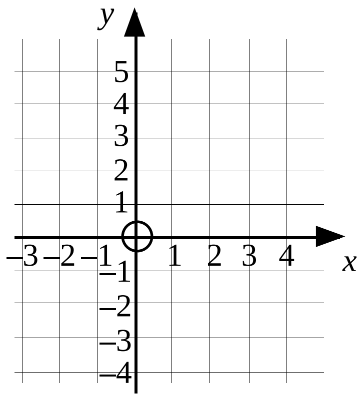
(f)



Checkup for Linear Relationships

1. (a) Given the two points A(2,–1) and B(4,7), calculate the gradient of the line AB.

   (b) Repeat for the line joining C(–7,2) and D(1,–4).

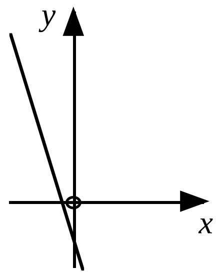


2. Make a copy of this coordinate diagram and draw the line $y = 2x - 2$.

3. Sketch the line $y = \frac{1}{2}x + 1$.

4. Sketch the line $y = -x - 1$.

5. Which of the following is most likely to be the equation of the line shown opposite?



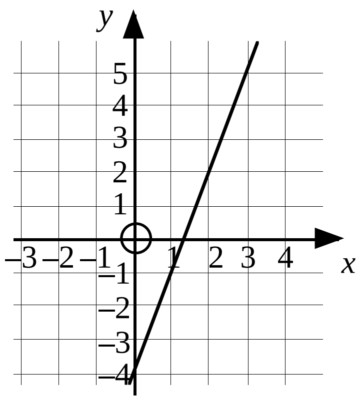
   (a) $y = 3x - 2$ (b) $y = -3x - 2$

   (c) $y = \frac{1}{3}x - 2$ (d) $y = \frac{1}{3}x + 2$

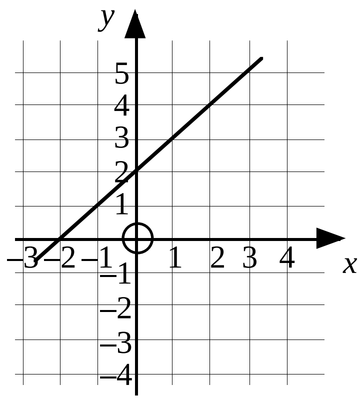
   (e) $y = -3x + 2$ (f) $y = -\frac{1}{3}x - 2$

6. Determine the equations of the following four lines in the form $y = ax + b$:

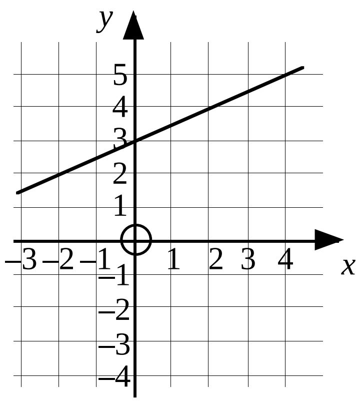
(a)



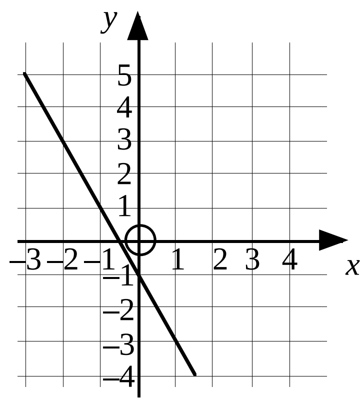
(b)



(c)



(d)



ALGEBRAIC OPERATIONS

By the end of this set of exercises, you should be able to

(a) multiply algebraic expressions involving brackets

(b) factorise algebraic expressions

(c) factorise trinomial expressions

ALGEBRAIC OPERATIONS

A. Multiplying Algebraic Expressions Involving Brackets

Exercise 1

1. Write these without brackets:

(a) $6(x + 2)$ (b) $3(a + 1)$ (c) $5(y - 4)$ (d) $7(t - 1)$
(e) $10(x - 10)$ (f) $2(2 + x)$ (g) $3(4 + y)$ (h) $6(5 - w)$
(i) $8(1 - c)$ (j) $15(2 - h)$ (k) $3(x + y)$ (l) $9(a - c)$
(m) $4(2 - x)$ (n) $11(e - f)$ (o) $1(1 - y)$ (p) $1(y - 1)$

2. Remove the brackets:

(a) $3(2x + 4)$ (b) $2(4a + 3)$ (c) $5(1 + 2y)$ (d) $6(3 - 3x)$
(e) $7(2w - 4)$ (f) $c(x + 5)$ (g) $d(v + 3)$ (h) $g(h - 1)$
(i) $s(r - 4)$ (j) $m(n + 10)$ (k) $x(v + w)$ (l) $a(x + r)$
(m) $x(a - y)$ (n) $a(a + b)$ (o) $r(r - s)$ (p) $r(r - 1)$
(q) $a(1 - a)$ (r) $x(x - 8)$ (s) $x(x + 3y)$ (t) $w(3w - 1)$
(u) $x(5x - 3)$ (v) $a(7x - 5a)$ (w) $m(4m + 8n)$ (x) $v(27 - 2v)$

3. Multiply out the brackets:

(a) $2(x + y + 4)$ (b) $7(x + y + 1)$ (c) $5(x - y - 6)$
(d) $6(x + 2y + 5)$ (e) $10(4x - y + z)$ (f) $9(6a - 2b + 1)$
(g) $x(3x + 5y + z)$ (h) $2a(3a - 4b + c)$ (i) $s(s^2 + 3)$
(j) $x(x^2 + 1)$ (k) $y(y^2 - 1)$ (l) $c(c^2 - 6)$
(m) $w(w^2 + w)$ (n) $a(a^2 - a)$ (o) $x(x^3 - 2x^2)$

Exercise 2A

1. Multiply out these brackets:

(a) $(x + 1)(x + 5)$ (b) $(x + 2)(x + 3)$ (c) $(x + 5)(x + 6)$
(d) $(x + 3)(x + 7)$ (e) $(x + 4)(x + 4)$ (f) $(x + 1)(x + 1)$
(g) $(a + 1)(a + 8)$ (h) $(s + 11)(s + 10)$ (i) $(w + 4)(w + 100)$

2. Multiply:

(a) $(x - 3)(x - 1)$ (b) $(x - 4)(x - 2)$ (c) $(x - 7)(x - 8)$
(d) $(a - 2)(a - 5)$ (e) $(b - 7)(b - 7)$ (f) $(c - 3)(c - 2)$
(g) $(v - 10)(v - 10)$ (h) $(w - 6)(w - 3)$ (i) $(z - 1)(z - 1)$

3. Multiply:

(a) $(x + 5)(x + 1)$ (b) $(c - 4)(c - 2)$ (c) $(s - 6)(s + 3)$
(d) $(a - 7)(a - 5)$ (e) $(v + 9)(v + 9)$ (f) $(q - 6)(q + 2)$
(g) $(r + 6)(r - 2)$ (h) $(w - 8)(w + 8)$ (i) $(x + 1)(x - 1)$
(j) $(d - 3)(d - 3)$ (k) $(a - 6)(a + 11)$ (l) $(z - 10)(z + 11)$

4. Multiply:

(a) $(2x + 3)(2x - 3)$ (b) $(5c - 1)(5c + 1)$ (c) $(2s - 1)(2s + 3)$
(d) $(2a - 3)(2a - 1)$ (e) $(v + 1)(4v - 3)$ (f) $(3q - 4)(2q + 3)$
(g) $(4r - 2)(5r + 3)$ (h) $(4w - 5)(2w + 5)$ (i) $(10x + 1)(10x - 1)$
(j) $(2 - d)(1 - d)$ (k) $(4 - p)(3 + 2p)$ (l) $(1 - 3p)(1 - 2p)$

5. Multiply out:

(a) $(x + 2)^2$ (b) $(y + 4)^2$ (c) $(z + 3)^2$ (d) $(t + 10)^2$
(e) $(x - 1)^2$ (f) $(y - 6)^2$ (g) $(z - 2)^2$ (h) $(t - 8)^2$
(i) $(a + b)^2$ (j) $(g + h)^2$ (k) $(r - s)^2$ (l) $(e - f)^2$
(m) $(3x + 1)^2$ (n) $(4x - 3)^2$ (o) $(x + 3y)^2$ (p) $(a - 4b)^2$
(q) $(4a + b)^2$ (r) $(5c + d)^2$ (s) $(5p + 2q)^2$ (t) $(2x - 3y)^2$

Exercise 2B

Multiply out the brackets and simplify:

1. $(x + 1)(x^2 + 3x + 1)$ 2. $(x + 2)(x^2 - 4x + 1)$
3. $(w - 3)(w^2 + w - 2)$ 4. $(z - 1)(z^2 - 5z - 1)$
5. $(v + 2)(2v^2 + v + 5)$ 6. $(a - 5)(5a^2 - 10a - 20)$
7. $(m + 2)^3$ 8. $(n - 1)^3$
9. $(x + {}^1/_x)^2$ 10. $(x - {}^1/_x)^2$

B. Factorising Algebraic Expressions – The Common Factor

Exercise 3

1. Factorise the following by taking out the common factors:

(a) $4a + 4b$ (b) $7v + 7w$ (c) $3x - 3y$ (d) $6c - 6d$
(e) $2r + 4s$ (f) $9m - 12n$ (g) $av + aw$ (h) $pq - pr$
(i) $bx + b$ (j) $ax^2 + a$ (k) $x^2 + dx$ (l) $y^2 - yz$
(m) $a^2 + a$ (n) $t^2 - t$ (o) $h^3 + h^2$ (p) $m^3 - m^2$
(q) $ab + bt$ (r) $mn - nr$ (s) $8x + 12y$ (t) $35p - 21q$
(u) $2a^2 + 8ab$ (v) $12ab - 9ac$ (w) $pqr + pqs$ (x) $8c^2 - 2c$

2. Factorise:

(a) $am - bm$ (b) $20 - 5w$ (c) $d - d^2$ (d) $yz + z$
(e) $pr - pu$ (f) $2mn + mp$ (g) $6cd - 4ce$ (h) $9pq - 12pr$
(i) $8a^2 + 6a$ (j) $15x^2 - 6xy$ (k) ${}^1/_2x + {}^1/_2y$ (l) $pq + {}^1/_2sq^2$
(m) $10a^2b + 8ab^2$ (n) ${}^1/_2 + {}^1/_2x$ (o) ${}^1/_2v - {}^3/_2$ (p) $2\pi rh + 2\pi r^2$
(q) $6a + 3b - 12c$ (r) $mn - mp + m^2$ (s) $3x^2 - 2xy + 6x$ (t) $25x^2 - 5x^2y$

C. Difference of Two Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Exercise 4

1. Factorise:

(a) $x^2 - y^2$ (b) $p^2 - q^2$ (c) $d^2 - e^2$ (d) $x^2 - 3^2$

(e) $y^2 - 4^2$ (f) $t^2 - 5^2$ (g) $5^2 - t^2$ (h) $9^2 - q^2$

(i) $1 - v^2$ (j) $x^2 - 4$ (k) $k^2 - 25$ (l) $n^2 - 36$

(m) $d^2 - 100$ (n) $e^2 - 121$ (o) $144 - y^2$ (p) $49 - x^2$

(q) $x^2 - 1$ (r) $1 - y^2$ (s) $81 - a^2$ (t) $10000 - b^2$

2. Factorise:

(a) $9a^2 - 4$ (b) $4b^2 - 25$ (c) $16c^2 - 1$ (d) $25d^2 - 36$

(e) $9e^2 - 16$ (f) $25f^2 - 81$ (g) $4g^2 - h^2$ (h) $j^2 - 25k^2$

(i) $64m^2 - 49n^2$ (j) $4p^2 - 9q^2$ (k) $81r^2 - 1$ (l) $1 - 64s^2$

(m) $121 - 16t^2$ (n) $100u^2 - 121v^2$ (o) $10000w^2 - 1$ (p) $25x^2 - 49y^2$

3. Factorise these, by taking out the common factor first:

(a) $2a^2 - 18$ (b) $5b^2 - 5$ (c) $6c^2 - 54$ (d) $4d^2 - 16$

(e) $7e^2 - 7g^2$ (f) $6p^2 - 24q^2$ (g) $10x^2 - 90y^2$ (h) $12u^2 - 12v^2$

(i) $am^2 - an^2$ (j) $ka^2 - 25kb^2$ (k) $nr^2 - 81nq^2$ (l) $d^3 - 49d$

(m) $64b - b^3$ (n) $2u^3 - 32u$ (o) $12w^3 - 27w$ (p) $11x^5 - 11x^3$

D. Trinomial Expressions

Exercise 5

Factorise the expressions:

1. $x^2 + 3x + 2$ 2. $x^2 + 5x + 6$ 3. $x^2 + 2x + 1$

4. $y^2 + 6y + 5$ 5. $y^2 + 11y + 10$ 6. $y^2 + 8y + 7$

7. $v^2 + 9v + 20$ 8. $v^2 + 7v + 10$ 9. $v^2 + 6v + 8$

10. $w^2 - 2w + 1$ 11. $w^2 - 4w + 4$ 12. $w^2 - 6w + 9$

13. $a^2 - 3a + 2$ 14. $a^2 - 7a + 12$ 15. $a^2 - 8a + 7$

16. $c^2 - 13c + 42$ 17. $c^2 - 11c + 24$ 18. $c^2 - 10c + 9$

19. $s^2 + 12s + 36$ 20. $s^2 - 12s + 36$ 21. $s^2 + 14s + 49$

22. $z^2 - 14z + 49$ 23. $z^2 + 13z + 36$ 24. $z^2 - 13z + 36$

25. $b^2 + 37b + 36$ 26. $b^2 - 37b + 36$ 27. $b^2 - 18b + 81$

cont'd

28. $p^2 + 6p + 9$ 29. $p^2 - 7p - 8$ 30. $p^2 + 4p + 4$
31. $m^2 + 11m + 30$ 32. $m^2 + m - 12$ 33. $m^2 - m - 6$
34. $n^2 - 8n + 15$ 35. $n^2 + 3n - 10$ 36. $n^2 - 3n - 4$
37. $r^2 - 2r - 8$ 38. $r^2 + 5r - 6$ 39. $r^2 + 12r + 36$
40. $e^2 - 5e - 14$ 41. $e^2 + 7e + 12$ 42. $e^2 - e - 56$
43. $g^2 - 7g + 12$ 44. $g^2 - g - 6$ 45. $g^2 - g - 12$
46. $k^2 - 4k - 5$ 47. $k^2 + k - 6$ 48. $k^2 + 2k - 35$
49. $y^2 + 4y - 12$ 50. $y^2 + 3y - 18$ 51. $y^2 - 3y - 28$
52. $x^2 - 3x - 40$ 53. $x^2 - 2x - 15$ 54. $x^2 + 11x + 30$
55. $v^2 - 9v + 8$ 56. $v^2 + 5v - 24$ 57. $v^2 - 5v - 24$
58. $w^2 + 2w - 24$ 59. $w^2 - 2w - 24$ 60. $w^2 + 10w - 24$
61. $a^2 - 10a - 24$ 62. $a^2 + 23a - 24$ 63. $a^2 - 23a - 24$
64. $b^2 + 7b - 30$ 65. $b^2 - 4b - 45$ 66. $b^2 - 7b - 18$
67. $c^2 + 15c + 56$ 68. $c^2 - 15c + 54$ 69. $\mathrm{c}^2 + 18c + 81$
70. $d^2 - 12d - 28$ 71. $d^2 + 49d - 50$ 72. $d^2 - 51d + 50$
73. $a^2 + 2ab + b^2$ 74. $x^2 - 2xy + y^2$ 75. $p^2 - pq - 2q^2$

Exercise 6

Factorise these expressions:

1. $2x^2 + 7x + 3$ 2. $2y^2 + 5y + 3$ 3. $3w^2 + 7w + 2$
4. $10a^2 + 17a + 3$ 5. $6b^2 + 7b + 2$ 6. $6c^2 + 7c + 1$
7. $3d^2 + 14d + 15$ 8. $10m^2 + 19m + 6$ 9. $2p^2 - 7p + 3$
10. $12n^2 - 8n + 1$ 11. $2q^2 - 5q + 3$ 12. $6x^2 - 13x + 6$
13. $8s^2 - 14s + 5$ 14. $9r^2 - 24r + 16$ 15. $12g^2 - 23g + 10$
16. $3k^2 - 5k + 2$ 17. $3y^2 - 2y - 8$ 18. $3w^2 - 5w - 2$
19. $6u^2 - 5u - 6$ 20. $5v^2 + 4v - 1$ 21. $2x^2 + x - 1$
22. $3d^2 - 2d - 1$ 23. $8a^2 + 2a - 3$ 24. $12y^2 - 11y - 5$
25. $4p^2 - 11p + 6$ 26. $15 - 7\mathrm{x} - 2x^2$ 27. $5 + 11\mathrm{x} - 12x^2$
28. $1 - 8\mathrm{x} + 16x^2$ 29. $1 - 3\mathrm{x} - 18x^2$ 30. $4p^2 - 7pq - 2q^2$

Exercise 7 (Miscellaneous Examples on Factorisation)

Factorise FULLY:

1. $4x + 12y$
2. $a^2 – 81$
3. $w^2 + 10w + 25$
4. $y^2 – y$
5. $v^2 – v – 12$
6. $1 – b^2$
7. $u^2 + 12u + 36$
8. $ap – aq + ar$
9. $7x^2 – 28$
10. $w^2 – r^2$
11. $h^2 – 11h$
12. $x^2 – 2x+ 1$
13. $t^2 – 1$
14. $t^2 – t$
15. $a^2 – 2a – 3$
16. $3c^2 – 48$
17. $5d^2 – 20d$
18. $a^4 – a^3$
19. $2s^2 + 3s – 5$
20. $x^2 – 12x + 36$
21. $16y^2 + 8y + 1$
22. $49 – g^2$
23. $36 – 4r^2$
24. $14z – 7z^2$
25. $25 – 9g^2$
26. $2b^2 – b – 1$
27. $6x^2 + 7x – 3$
28. $11u^2 – 44v^2$
29. $21u^2 + 28v^2$
30. $25p^2 – 10p + 1$
31. $3m^2n – 6mn^2$
32. $1 – 2n + n^2$
33. $27 – 6s – s^2$
34. $3a^3 – 48a$
35. $8n^2 + 8n – 6$
36. $8n^2 – 8n + 2$
37. $5r^2 + 5r – 10$
38. $4w^2 + 14w – 8$
39. $7x – 63x^3$
40. $9x + 27x^3$
41. $xy^2 – xz^2$
42. $2e^2 – 11e – 21$
43. $x^4 – 1$
44. $2 – 4q + 2q^2$
45. $g^2 + gh – 6h^2$
46. $2k^2 + 3\pi Rk + \pi^2R^2$
47. $a^2 – a^6$
48. $k^4 + 2k^2 + 1$
49. $2a^4 – 2a^2 – 12$
50. $b^5 – 81b$
51. $3x^4 + 5x^2 – 2$
52. $9x^4 – 24x^2 + 16$
53. $2x^4 – x^2 – 3$
54. $1 – y^8$

Checkup for Algebraic Operations

1. Remove the brackets:

(a) $3(4x + 1)$	(b) $y(a - y)$	(c) $v(v - 1)$	(d) $7w(2w - 5)$
(e) $6(3x + 2y - 1)$	(f) $c(c^2 + c - 1)$	(g) $3d(4a + 3b)$	(h) $g(h^2 - g^2)$
(i) $6x(3x + 2y - 1)$	(j) $c^2(c^2 + c - 4)$	(k) $ab(3a + 4b)$	(l) $2pq(5 - q)$

2. Multiply out the brackets:

(a) $(x + 1)(x + 7)$	(b) $(x - 2)(x - 3)$	(c) $(x + 5)(x - 6)$
(d) $(x - 3)(x + 9)$	(e) $(x + 1)^2$	(f) $(x - 2)^2$
(g) $(5x - 1)(4x + 7)$	(h) $(2x - 1)(6x - 3)$	(i) $(2x - 4)(3x + 1)$
(j) $(2x - 3)^2$	(k) $(x - 2)(4x^2 - 3x + 2)$	(l) $(x - 3)^3$

3. Factorise fully:

(a) $9m - 9n$	(b) $6a - 15b$	(c) $y - y^2$
(d) $14p^2 + 6q$	(e) $3pr + pu$	(f) $4p^2 + 6pq - 2p$
(g) $6x + 30y - 15z$	(h) $9pq - 12pr$	(i) $r^2 - s^2$
(j) $81 - q^2$	(k) $16r^2 - 49$	(l) $2b^2 - 32$
(m) $20w^3 - 45w$	(n) $y^2 - 3y + 2$	(o) $a^2 - 7a - 30$
(p) $y^2 + y - 6$	(q) $24 + 10r - r^2$	(r) $x^2 - 14x + 49$
(s) $6p^2 - 17p + 12$	(t) $4x^2 + 4x + 1$	(u) $2q^2 - 2q - 144$
(v) $2x^2 + 3xy - 2y^2$	(w) $6a^4 + 2a^2 - 4$	(x) $5y^4 - 12y^2 - 9$

PROPERTIES OF THE CIRCLE

By the end of this set of exercises, you should be able to

(a) find the length of an arc of a circle

(b) find the area of a sector of a circle

(c) use the properties of of circles:
- relationship between tangent and radius
- angle in a semi-circle
- the interdependence of the centre, bisector of a chord and a perpendicular to a chord

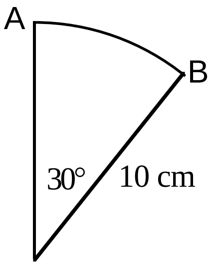
PROPERTIES OF THE CIRCLE

A. Finding the length of an arc

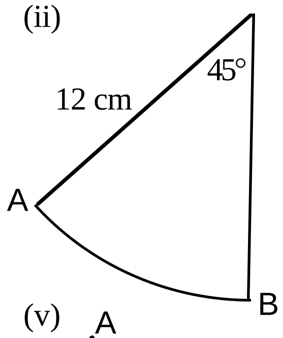
Exercise 1

1. In each diagram, calculate the length of the arc AB of the sector.

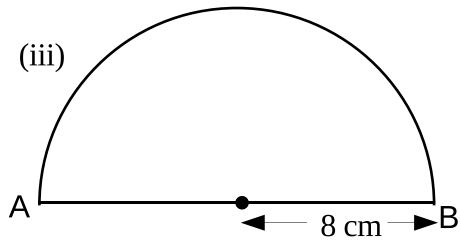
(i)



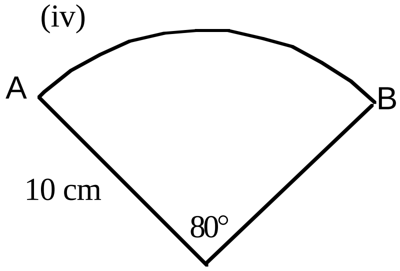
(ii)



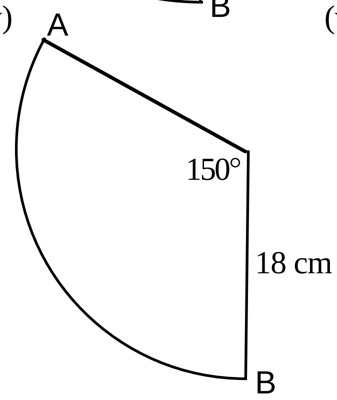
(iii)



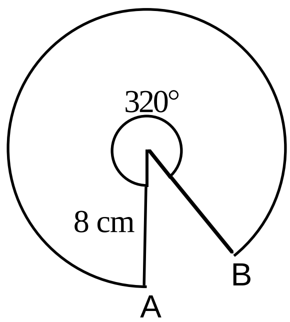
(iv)



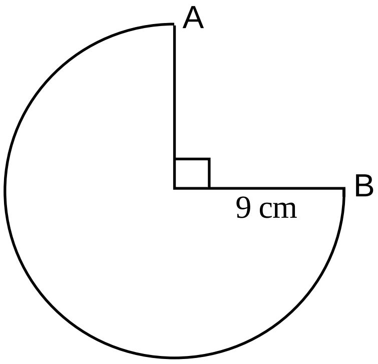
(v)



(vi)

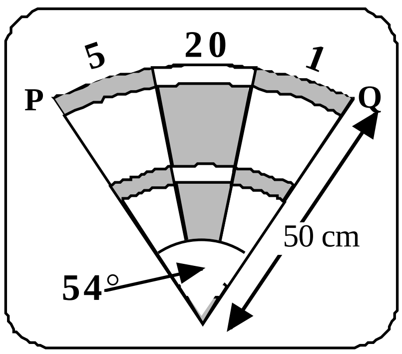


(vii)

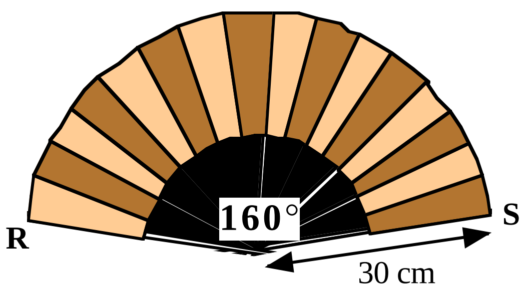


2. As the T.V. camera zoomed in on the dart board the sector involving the numbers 5, 20 and 1 was focused.

   Calculate the length of the arc PQ.

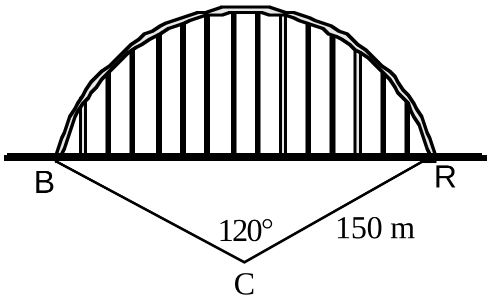


3.



Calculate the length of the curved edge RS of the fan which is in the shape of a sector of a circle.

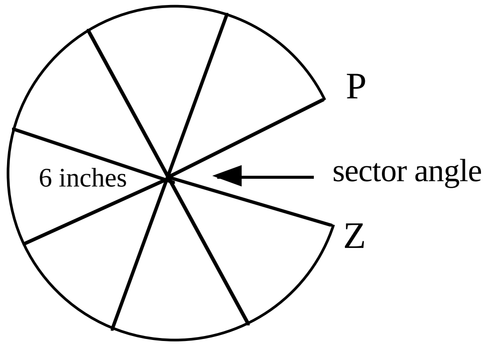
4. Calculate the length of the arch BR of the bridge which is the arc of a circle, centre C.



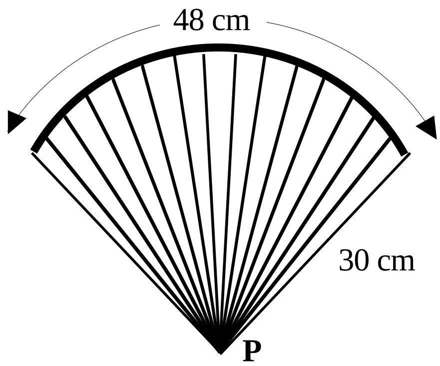
5. This circular pizza has been sliced into 8 pieces. Calculate:

(a) the size of the sector angle of one piece.

(b) the length of the **major** arc PZ.



6.



The lace edge of this fan is 48 cm long. It is an arc of a circle, centre P.

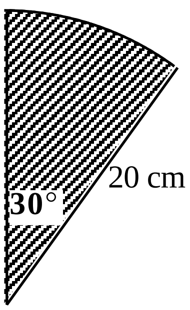
Calculate the size of the angle at P. (Answer to the nearest whole degree.)

B. Finding the area of a sector

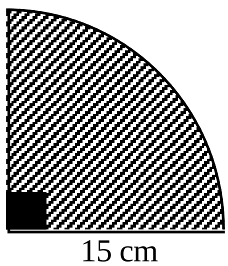
Exercise 2

1. Calculate the area of each sector (to the nearest square centimetre):

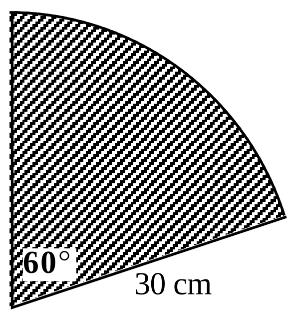
(a)



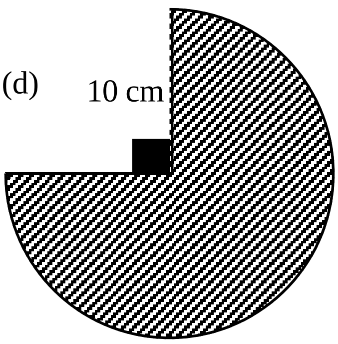
(b)



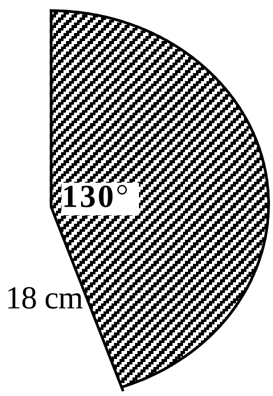
(c)



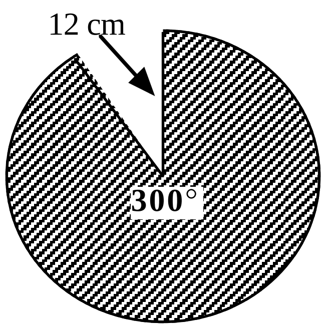
(d)



(e)



(f)

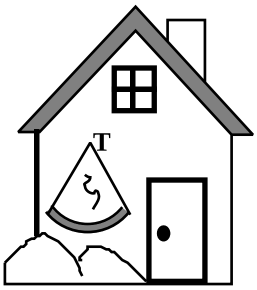


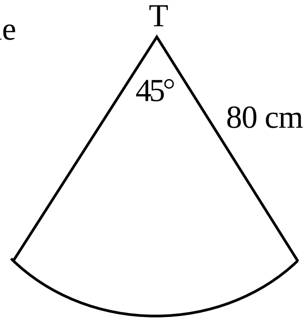
2\. This house has an unusually shaped living room window.

It is in the shape of a sector of a circle with radius 80 cm.

Unfortunately there is a crack in the glass and a new pane is required.

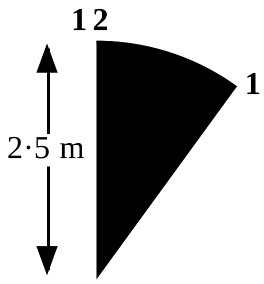
If the angle at T is 45°, calculate the area of glass to be replaced.

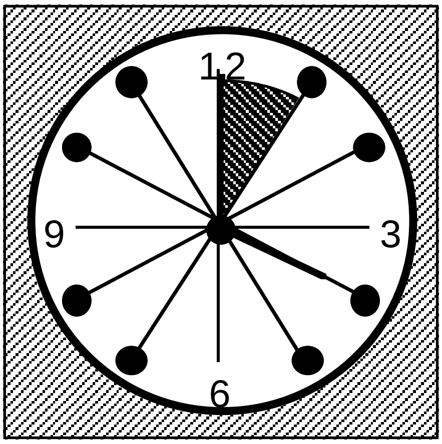




3\. The face of a large town clock was in need of repair.
Workmen were replacing the rusted sector between the numbers 12 and 1 on the clock face.

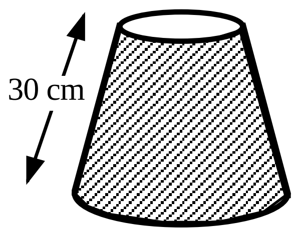
Calculate the area of this sector.

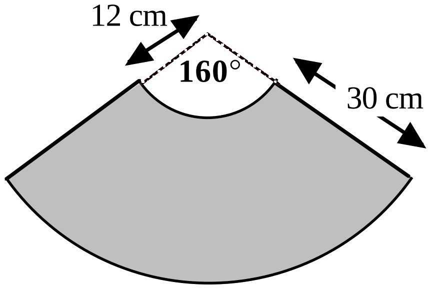




4\. A light shade is made up from the sector of a large circle with a smaller sector removed.

Calculate the area of the shade.



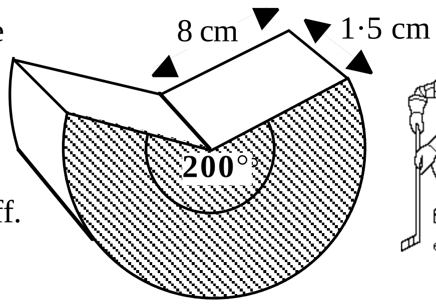


5\. A <u>cylindrical</u> ice hockey puck of radius 8 cm and height 1·5 cm hits a goal post with such force that it splits, leaving a perfect sector as shown.

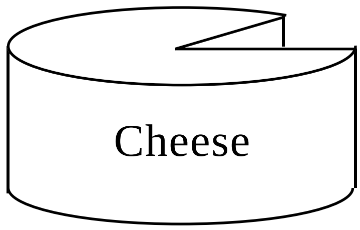
Calculate:

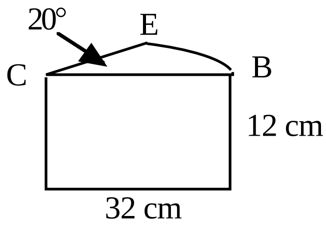
(a) the shaded area.

(b) the volume of the <u>smaller</u> part which broke off.



6. A wedge of cheese is cut from a large circular block of radius 32 cm and height 12 cm. For the wedge, the angle at C, the centre, is 20°.



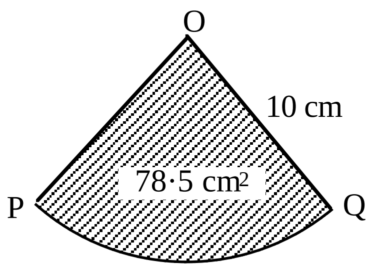


Calculate:

(a) the area of the sector BCE.

(b) the volume of the wedge of cheese.

7. The area of this sector is $78 \cdot 5\ \text{cm}^2$ and the radius of the circle from which it has been cut is 10 cm.

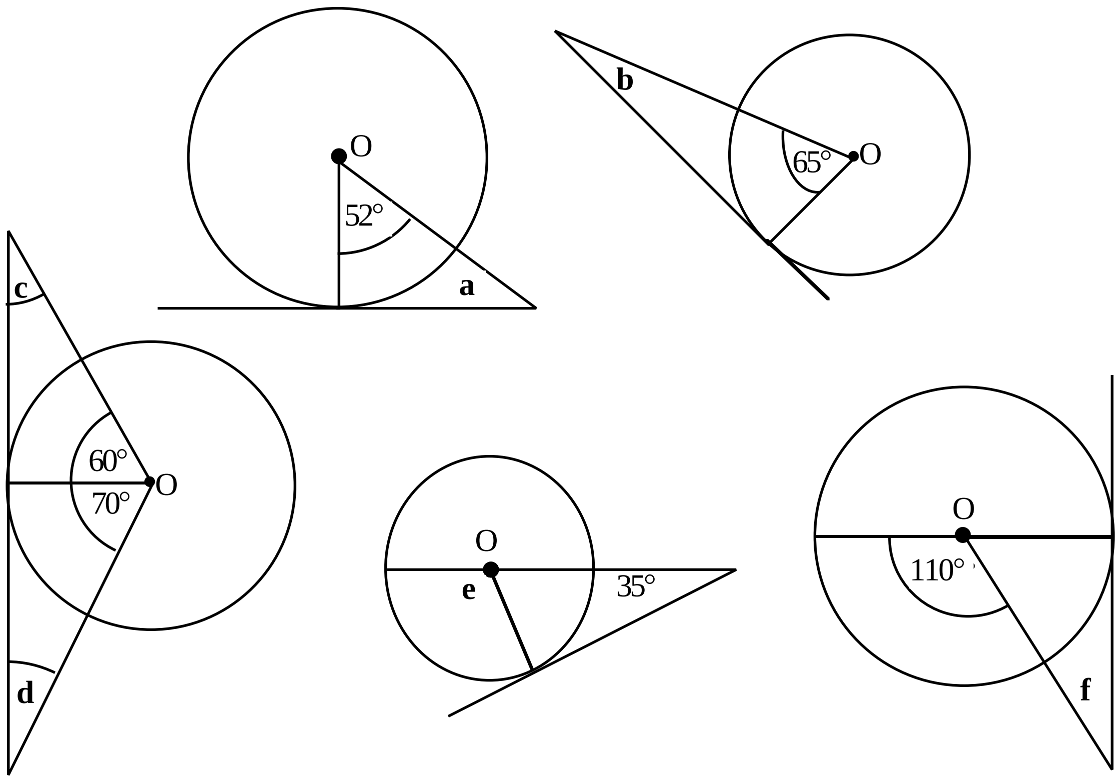


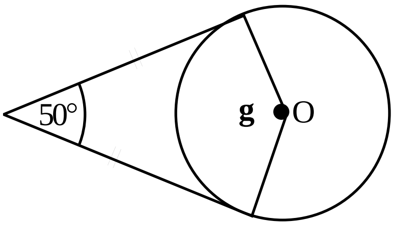
Calculate the size of angle POQ.

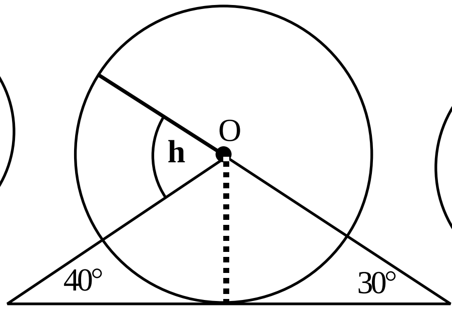
C. The relationship between tangent and radius

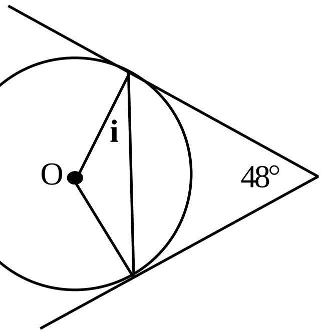
Exercise 3

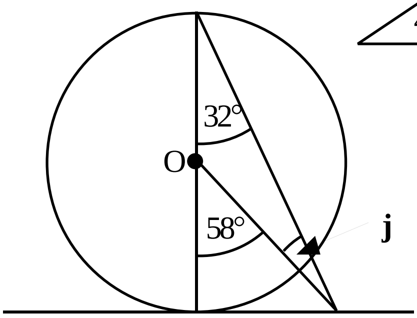
1. Copy the diagrams below and fill in the sizes of the angles marked with a letter.

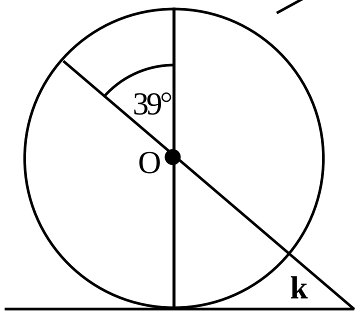










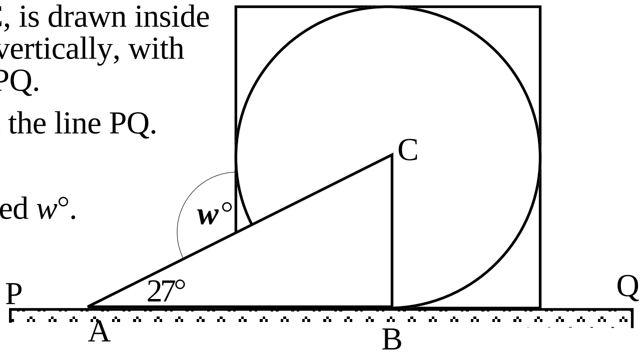


2. The largest possible circle, centre C, is drawn inside a square. The circle and square sit vertically, with one edge on the horizontal surface PQ.

Triangle ABC is drawn with AB on the line PQ.

Angle CAB = 27°

Calculate the size of the angle marked w°.

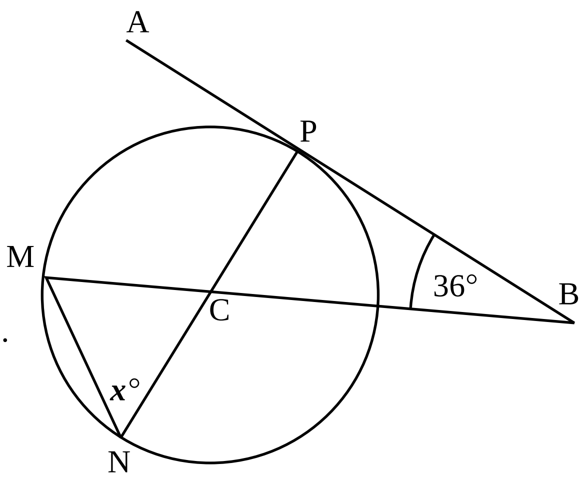


3. AB is a tangent to the circle with centre C.

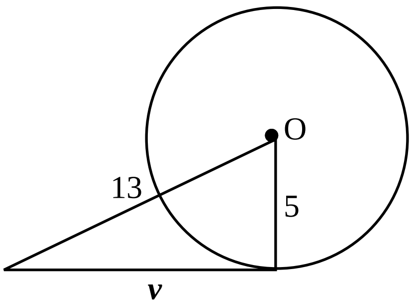
It meets the circle at the point P.

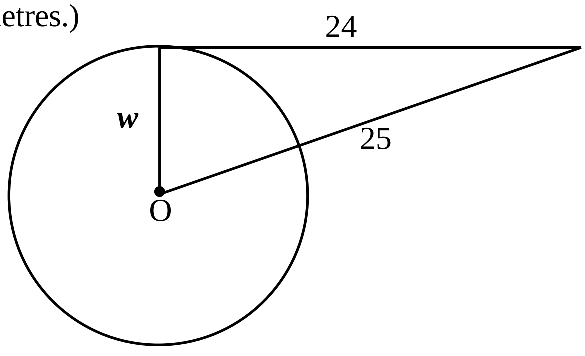
Angle CBP = 36°.

Calculate the value of the angle marked x°.

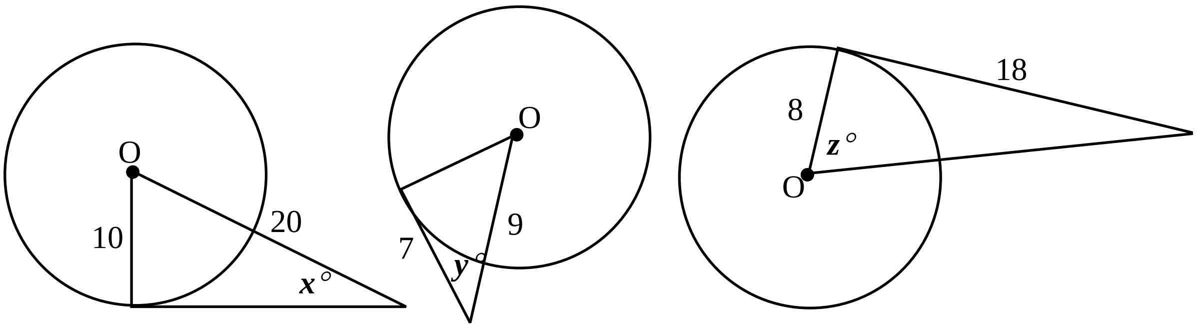


4. Calculate v and w. (The lengths are in centimetres.)

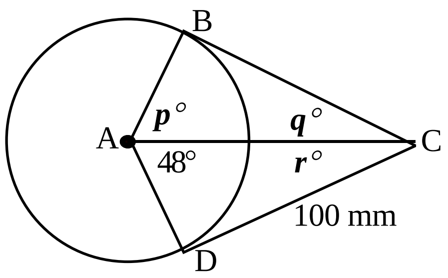




5. Calculate the sizes of the angles marked x, y and z correct to the nearest degree. (The lengths are in centimetres.)



6.



ABCD is a tangent kite.

Write down:

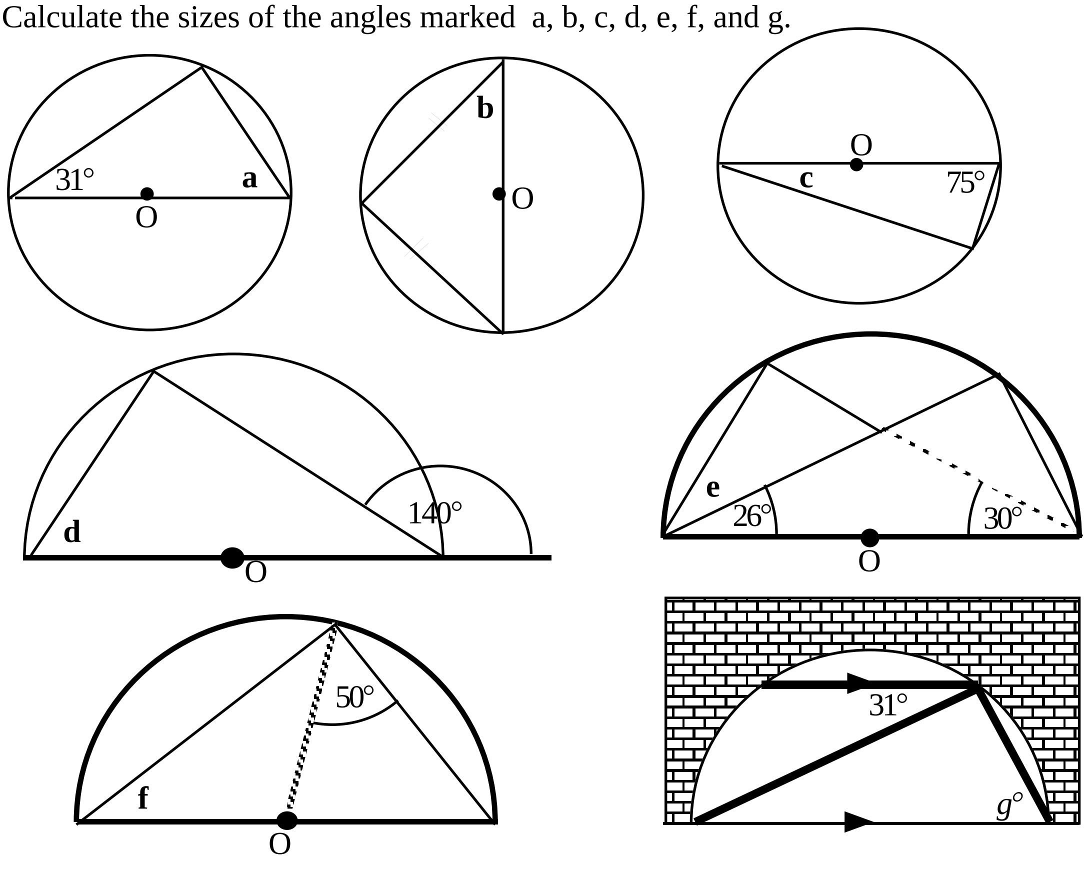
(a) the length of BC.

(b) the values of p, q and r.

D. Angle in a semi-circle

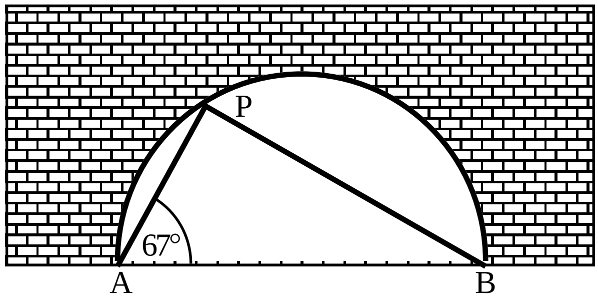
Exercise 4

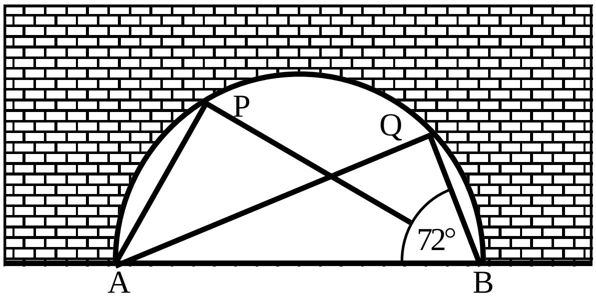
1. Calculate the sizes of the angles marked a, b, c, d, e, f, and g.



2. The semi-circular arch of a bridge is strengthened by a triangular metal structure as shown.

(a) Calculate the size of ∠ABP. 140°

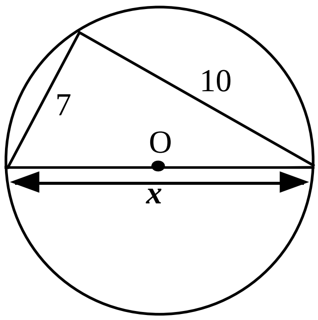




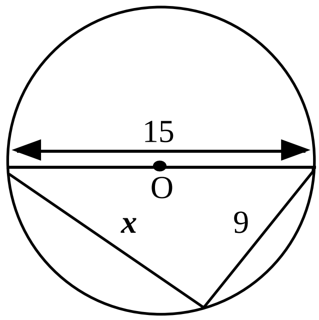
(b) A second triangular structure is added. Calculate the size of ∠PAQ.

3. In the two diagrams below, calculate *x*, correct to 1 decimal place.

(a)

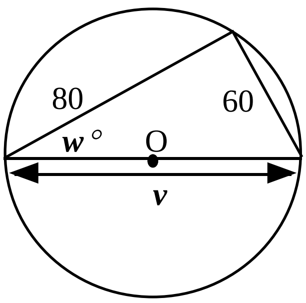


(b)

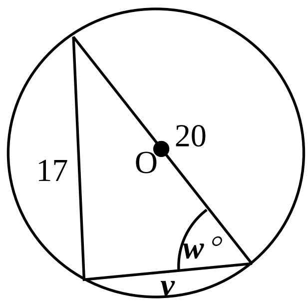


4. In these diagrams, calculate *v* and *w* correct to 1 decimal place.

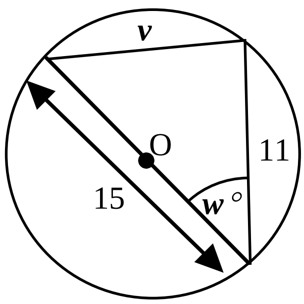
(a)



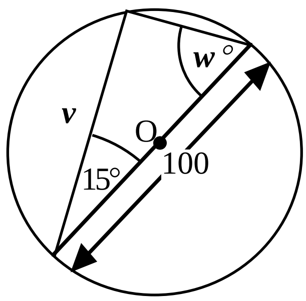
(b)



(c)



(d)

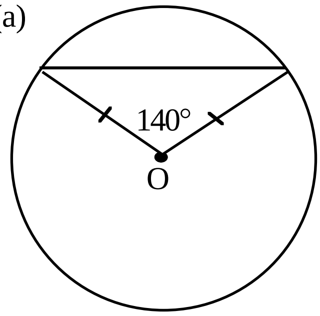


E. The interdependence of the centre, bisector of a chord and a perpendicular to a chord

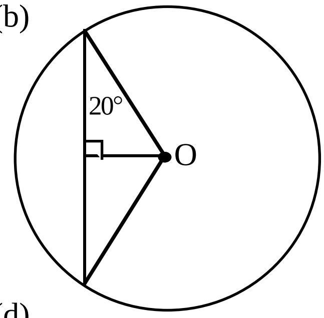
Exercise 5

1. Copy the diagrams and fill in all the angles.

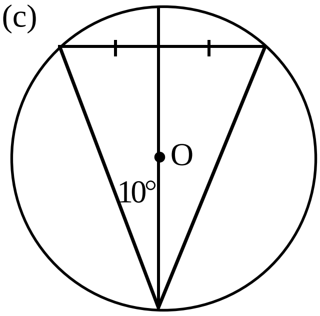
(a)



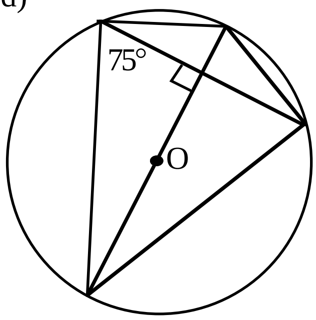
(b)



(c)

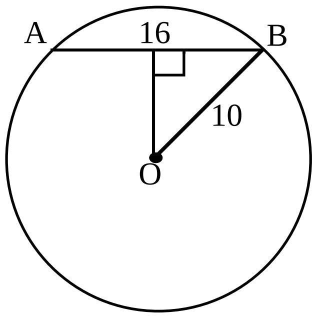


(d)

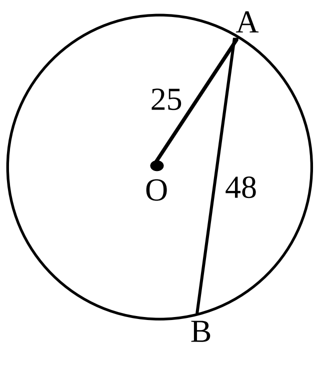


2. Calculate the distance from O to chord AB in each case. (All lengths are in centimetres.)

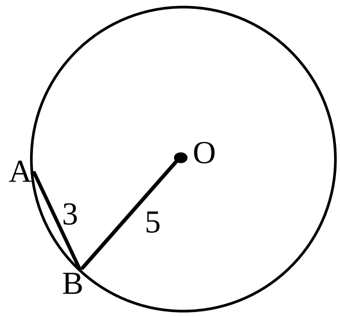
(a)



(b)

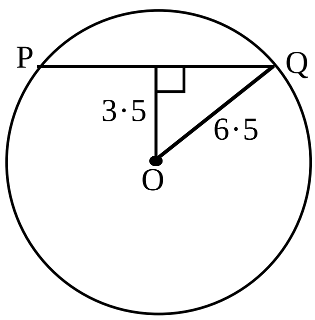


(c)

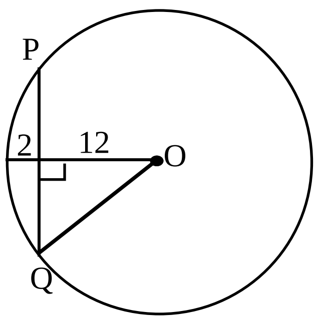


3. Calculate the length of the chord PQ in each case.

(a)

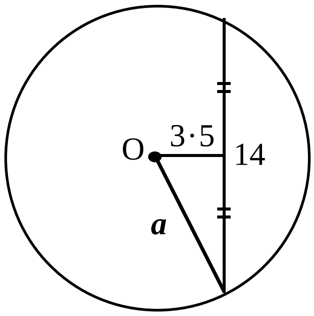


(b)

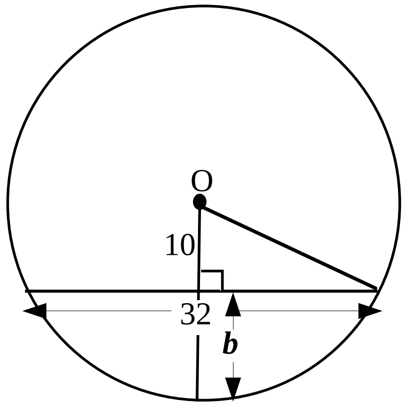


4. Calculate the value of the letters *a*, *b*, *c*, and *d*.

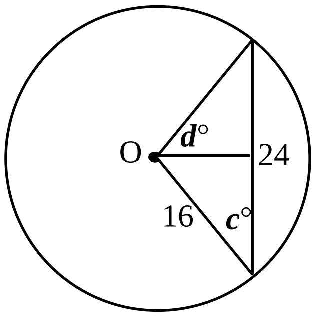
(i)



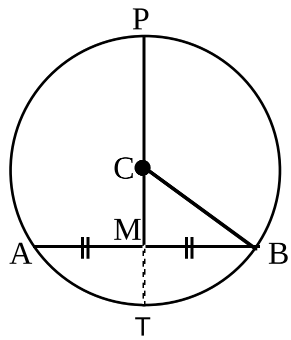
(ii)



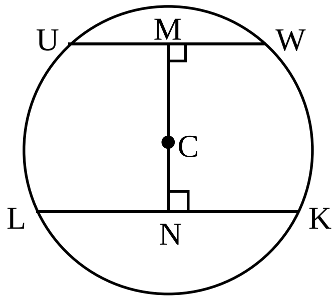
(iii)



5. Given that the radius of the circle is 25 cm and AB = 48 cm, calculate the length of the line MT.



6.

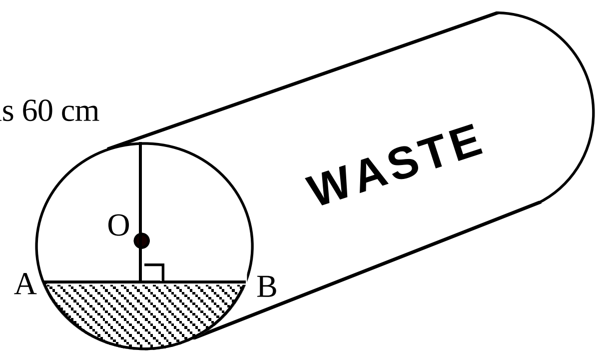


The diameter of the circle is 100 cm. UW = 62 cm and LK = 72 cm. and UW is parallel to LK.

Calculate the length of MN.

7. The diameter of a tank of waste product is 60 cm and the depth of the sludge is 25 cm.

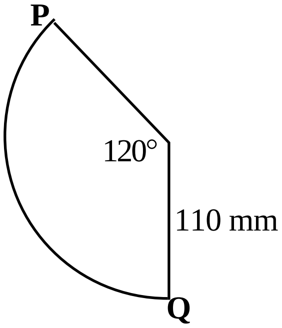
Calculate the width AB of the surface of the waste sludge.



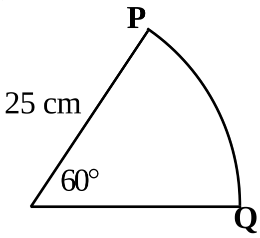
Checkup for Properties of a Circle

1. Calculate the length of the arc PQ of the sector.

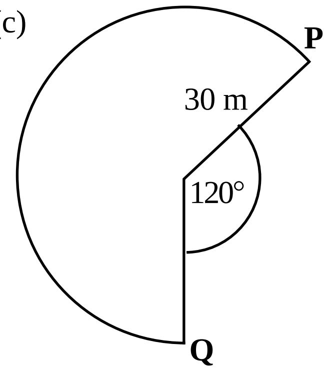
(a)



(b)

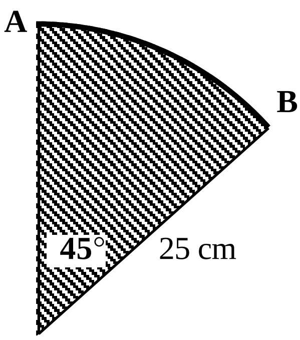


(c)

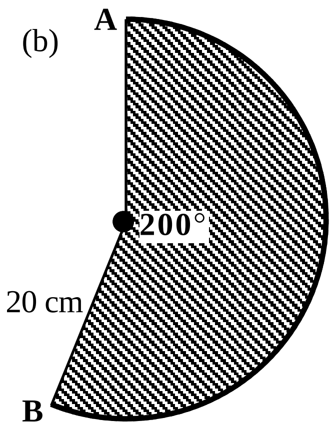


2. Calculate the area of each shaded sector AB:

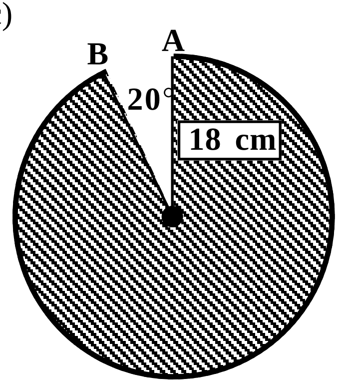
(a)



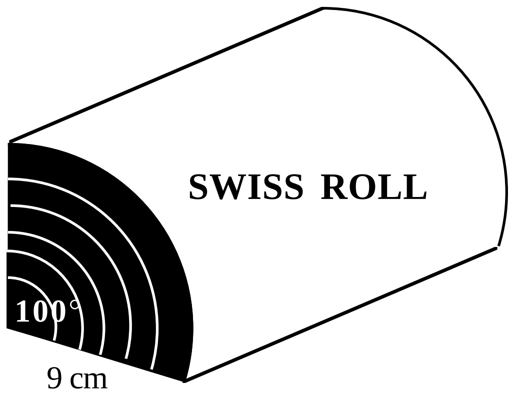
(b)



(c)



3.



Part of a Swiss Roll sponge is shown.

The chocolate sector has a radius of 9 cm and the sector angle is 100°.

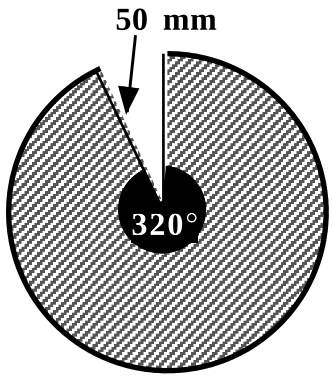
Calculate:

(a) The length of the arc around the chocolate sector.

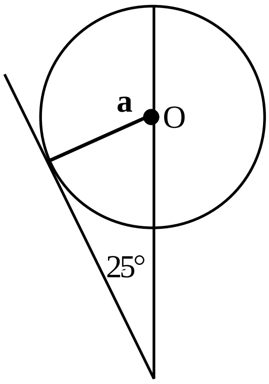
(b) The area of the chocolate sector.

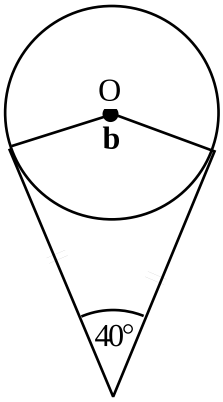
4. The safety guard on a circular saw has a sector angle of 320° and the radius of the blade is 50 mm.

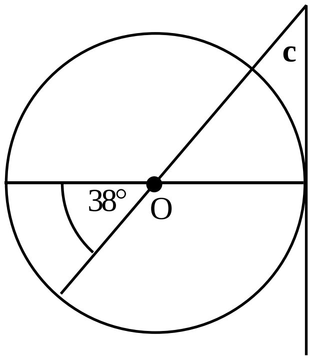
Calculate the area of the blade which is exposed, to the nearest mm^2.



5. Copy the diagrams below and fill in the sizes of the angles *a*, *b* and *c*.

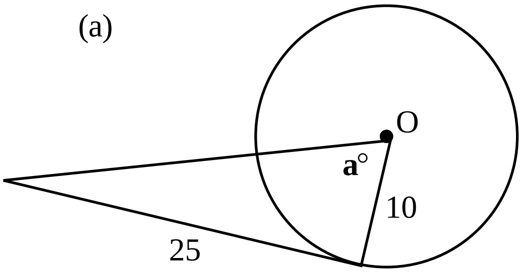




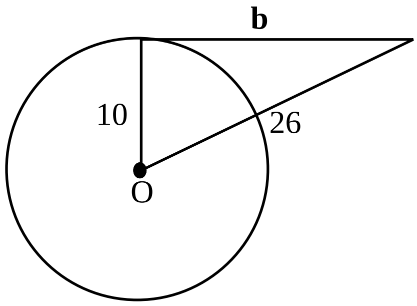


6. Calculate the value of a, b, c, d, and e. (All lengths in centimetres.)

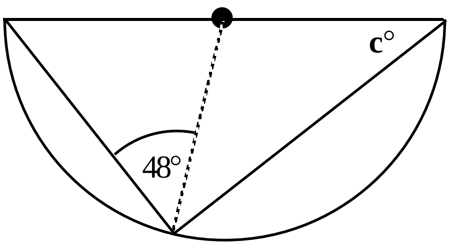
(a)



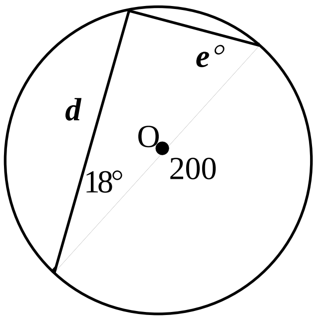
(b)



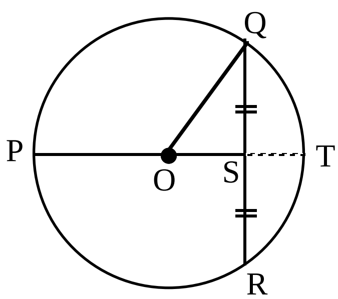
(c)



(d)



7. Given that the radius of the circle is 25 cm and QR = 48 cm, calculate the length of the line ST.



SPECIMEN ASSESSMENT QUESTIONS

1. Hans Segers deposited £500 in a Building Society for three years, leaving the interest to be added to his account each year. The annual rate of interest dropped from 5% in the first year to 4% in the second year and 2% in the third year.
   How much money was in his account after 3 years?

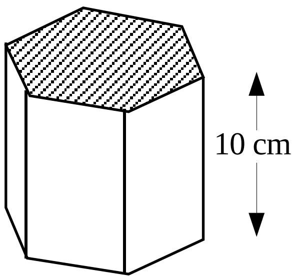
2. The value of a computer depreciates by 5% per annum.
   What is a £1500 computer worth after 20 years?
   Give your answer correct to one significant figure.

3. The Thomson’s bought an apartment in Spain for six million pesetas.
   It appreciated in value for the next three years by 10% in year 1, by 12·5% in year 2 and by 25% in year 3.
   How much was the apartment worth when they sold it after the three years?
   Give your answer correct to three significant figures.

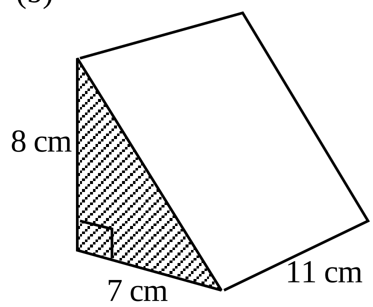
4. Mrs. Healey bought a cooker for £650 in 1998.
   One year later, the same cooker in the same shop was priced at £540.
   Calculate the percentage drop in price.
   Give your answer correct to two significant figures.

5. Calculate the volumes of the following prisms:

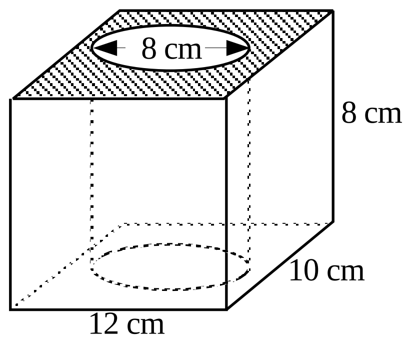
(a)



(b)

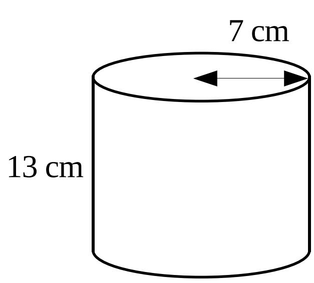


(c)

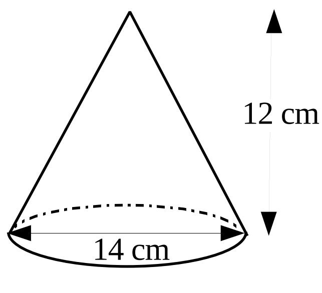


6. Calculate the volumes of the following shapes:

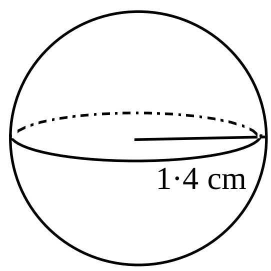
(a)



(b)

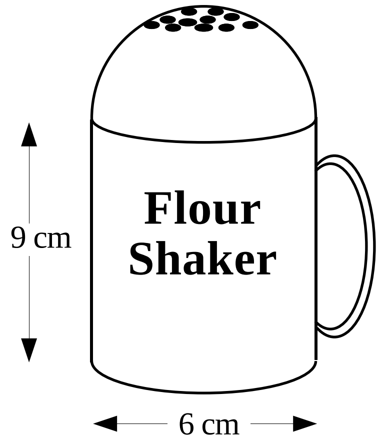


(c)



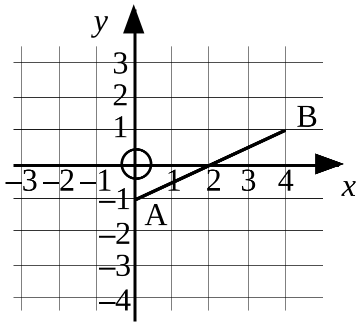
Vol $_{(cylinder)} = \pi r^2 h$ Vol $_{(cone)} = {}^1/_3\pi r^2 h$ Vol $_{(sphere)} = {}^4/_3\pi r^3$

7. Calculate the volume of this flour shaker which consists of a cylinder as base and a hemisphere as lid.

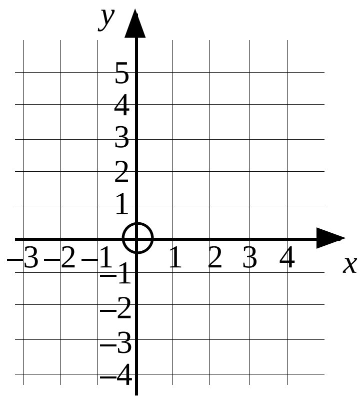


8. (a) Determine the gradient of the line AB shown on the diagram opposite.

(b) Determine the gradient of the line joining the two points P(–1,–4) and Q(–2,3)



9.

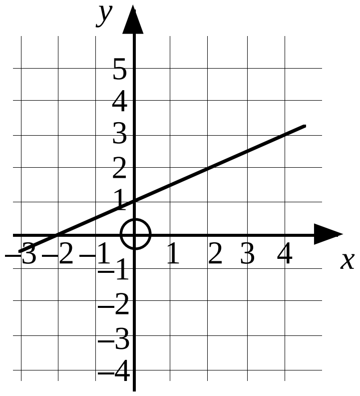


(a) Make a copy of this coordinate diagram on squared paper and draw the line $y = 3x - 2$

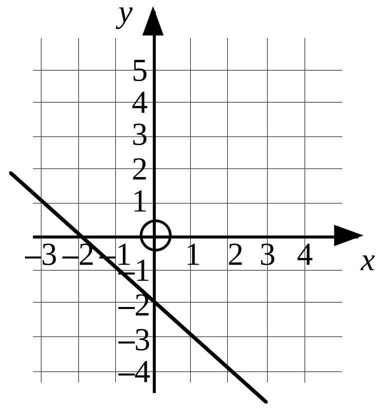
(b) Draw the line $y = -2x + 1$ on squared paper on a separate diagram.

10. Determine the equations of the following two lines:

(a)



(b)



11. Remove the brackets:

(a) $7(3x - 2)$ (b) $3v(2 - 9v)$ (c) $6(x - 5y + 2)$

(d) $2x(x^2 - x + 1)$ (e) $6p(7q + 2p)$ (f) $2x^2y(3x - y)$

(g) $(k - 3)(2k + 7)$ (h) $(3a - 5)^2$ (i) $(3x - 2)(x^2 + 5x - 1)$

12. Factorise fully:

(a) $9a - 21b$ (b) $vx + vy$ (c) $\pi m^2 - 2\pi m$

(d) $4d^2 - 9e^2$ (e) $2x^2 - 162$ (f) $t - t^3$

(g) $x^2 + 7x + 12$ (h) $a^2 - 9a + 18$ (i) $6y^2 - 5y - 4$

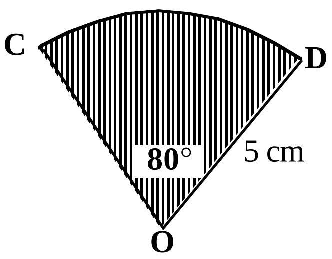
(j) $2b^2 - 6b - 20$ (k) $4p^2 - 11pq - 3q^2$ (l) $18 + 7w - w^2$

13. Calculate:

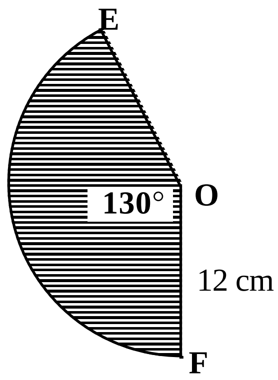
(i) the length of arc CD and EF.

(ii) the area of the shaded sector in each case.

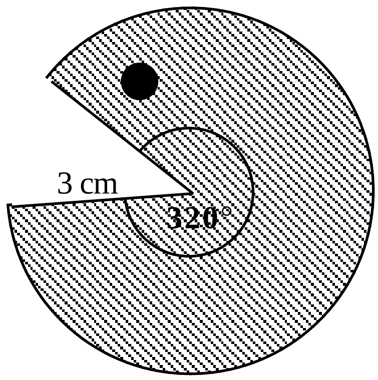
(a)



(b)



14.



Munchman is the leading character in a new computer game. He is in the shape of a sector of a circle with centre P.

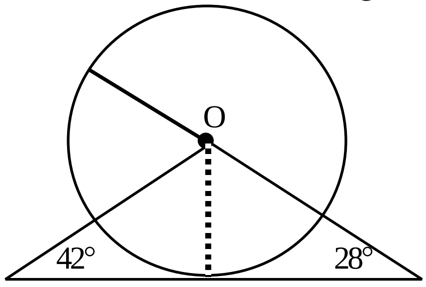
Calculate:

(a) his perimeter

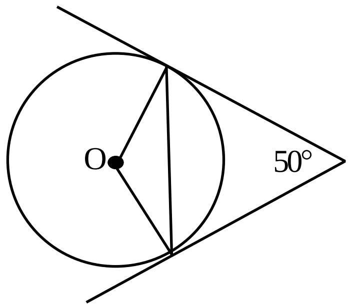
(b) his area, including his eye.

15. Make a neat sketch of these four diagrams and fill in all the angles.

(a)

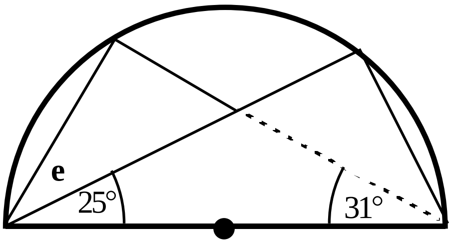


(b)

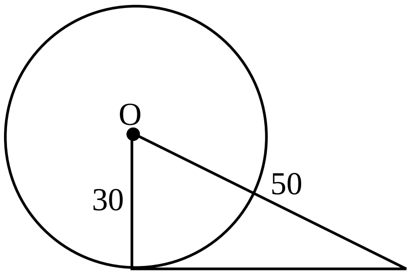


contd.

(c)

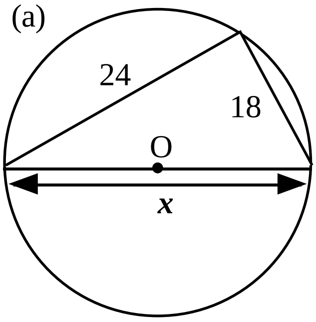


(d)

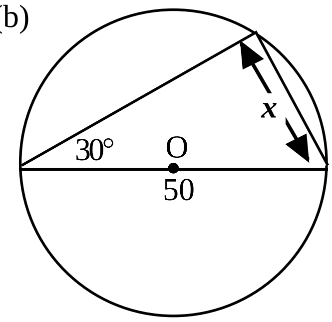


16. Calculate the length of the sides marked *x*.

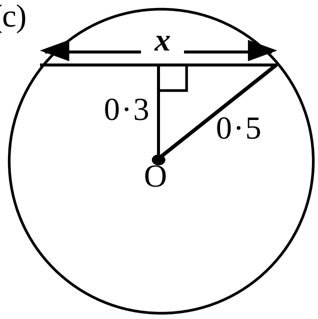
(a)



(b)

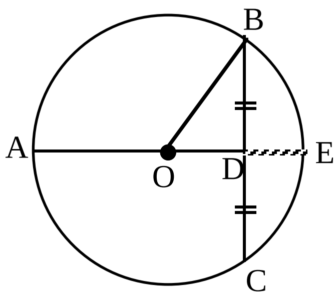


(c)



17. Given that the radius of the circle is 50 cm and BC = 96 cm,

calculate the length of the line DE.



ANSWERS

Calculations Involving Percentages

Exercise 1

1. (a) £12·75 (b) £21 (c) £1·10 (d) 68p (e) £9
(f) £48 (g) £7·20 (h) £4·80 (i) £3·50 (j) £7·60
(k) £1980 (l) 45p (m) £70 (n) £45 (o) £49·20
(p) £3·50 (q) £3·40 (r) 50p

2. (a) £30 (b) £80 3. (i) 42 (ii) 108 4. 482·5mm

5. 112·5g 6. (a) 76 (ii) 3724 7. 13440ft 8. 143·5cm 9. 2210

10. (a) £1308 (b) £1254 (c) £1281 (d) £1236 (e) £1245

11. 80% 12. 75% 13. 30% 14. 96·4%

Exercise 2

1. (a) £1389·15 (b) £703·04 (c) £52·02 2. £56·16

3. £623·70 4. £2803·50 5. £275·73

6. (a) Mrs. D £795·68 Mrs. E £795 (b) 3% per half year better as you get interest on the interest for rest of year.

7. £2929·64 8. 8 years

Exercise 3

1. £72600 2. £63559 3. £132848 4. £76098 5. £2160

6. £9000 7. £6055·20 8. 4·2% 9. (a) £86700 (b) 2%

10. (a) 5% (b) 8% 11. (a) 60% (b) 20% (c) 85% 12. £10000

Exercise 4

1. (a) 4000 (b) 10000 (c) 20 (d) 500 (e) 20000
(f) 2000 (g) 8000 (h) 7000 (i) 40000 (j) 500
(k) 10000 (l) 20000 (m) 7000000 (n) 60000000
(o) 40000000000 (p) 30

2. (a) 5200 (b) 25000 (c) 220 (d) 560 (e) 19000
(f) 2100 (g) 7700 (h) 6500 (i) 43000 (j) 450
(k) 78000 (l) 30000 (m) 6900000 (n) 56000000
(o) 39000000000 (p) 350

3. (a) 8180 (b) 24900 (c) 2220 (d) 5550 (e) 19600
(f) 2080 (g) 7680 (h) 6150 (i) 42600 (j) 4500
(k) 78200 (l) 29900 (m) 6890000 (n) 55800000
(o) 38700000000 (p) 35200000

4.

	(a)	(b)	(c)	(d)
	8	20	2	300
	8·3	24	1·5	350
	8·33	23·8	1·53	348

Exercise 5

1. (a) £2300 (b) £2000 (c) £4390

2. £950 3. £3670 4. £10000 5. £38000 6. £23700

7. (a) 47% (b) 41·7% (c) 90%

8. (a) 8% (b) 29% (c) 100%

Checkup for Calculations Involving Percentages

1. £33·28 2. £7400 3. £946 4. £700 5. £23800 6. £396
7. (a) £53531·25 (b) 39%
8. £13600

Volumes of Solids

Exercise 1

1. (a) 80 cm^3 (b) 75 cm^3 (c) 232 cm^3 (d) 572 cm^3
   (e) 64·4 cm^3 (f) 69·3 cm^3
2. (a) 350 cm^3 (b) 84 cm^3 (c) 675 cm^3 (d) 2040 cm^3
   (e) 960 cm^3 (f) 1243·44 cm^3
3. (a) 2009·6 cm^3 (b) 268·47 cm^3 (c) 255·125 cm^3
   (d) 1148·0625 cm^3 (e) 314 cm^3
4. (a) 78·5 litres (b) 98·91 litres (c) 69·08 litres
5. 384·65 cm^3
6. (a) 180000 cm^3 (b) 4019·2 cm^3 (c) 44
7. (a) 4 x 6 = 24 (b) 3 (c) 72 (d) 10897·92 cm^3
8. 16956 cm^3
9. 15260·4 cm^3

Exercise 2

1. (a) 565·2 cm^3 (b) 512·9 cm^3 (c) 230·8 cm^3 (d) 6699 cm^3 (e) 384·6 cm^3
2. 94·2 cm^3
3. (a) 24 cm (b) 2512 cm^3
4. (a) 2616·7 cm^3 + 36000 cm^3 = 38616·7 cm^3
   (b) 10173·6 cm^3 + 2543·4 cm^3 = 12717 cm^3
5. (a) 904·32 cm^3 (b) 18 seconds

Exercise 3

1. (a) 5572·5 cm^3 (b) 1149·8 cm^3 (c) 3260·1 cm^3
   (d) 14130 cm^3 (e) 588·7 cm^3
2. 7234·6 cm^3
3. (a) 1285·6 cm^3 (b) 718·0 cm^3
4. (a) 16746·66... + 16746·.. + 75360 = 108853·3 cm^3 (b) 108·9 litres
5. 564·15.. + 718·01... = 1282·2 cm^3
6. 454·3 cm^3

Checkup for Volumes of Solids

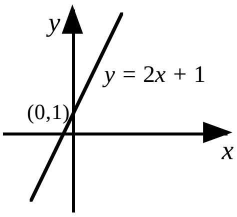
1. (a) $112{\cdot}5\ cm^3$ (b) $168\ cm^3$ (c) $185\ cm^3$
2. (a) $459\ cm^3$ (b) $1001\ cm^3$ (c) $536{\cdot}1\ cm^3$
3. (a) $2797{\cdot}7\ cm^3$ (b) $769{\cdot}3\ cm^3$ (c) $588{\cdot}7\ cm^3$
4. $678{\cdot}24\ cm^3 + 3391{\cdot}2\ cm^3 + 452{\cdot}16\ cm^3 = 4521{\cdot}6\ cm^3$

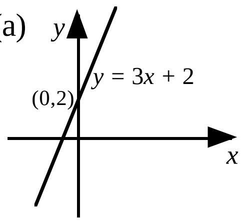
Linear Relationships

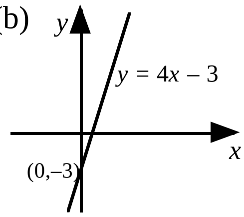
Exercise 1

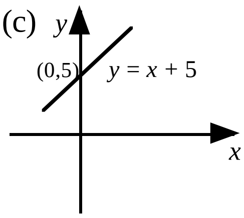
1. 2, 5, $\frac{1}{2}$, $\frac{2}{3}$
2. (a) 4 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
3. (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ (c) 6 (d) 3
4. slopes downwards if gradient is negative as you move from left to right
5. –1, –4, $-\frac{1}{3}$, $-\frac{3}{4}$
6. (a) –1 (b) –2 (c) –3 (d) $\frac{1}{3}$ (e) $-\frac{2}{3}$
   (f) 1 (g) –3 (h) $\frac{1}{2}$ (i) $-\frac{1}{3}$
7. (a) sketch showing vertical line. (b) gradient doesn't exist (error)
   (c) gradient of a vertical line does not exist.

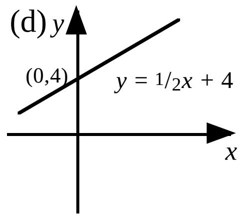
Exercise 2

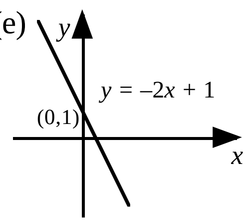
1.


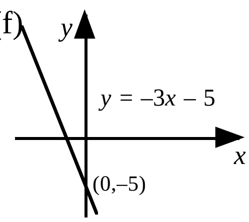
2. (a)


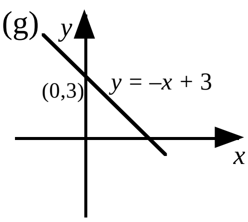
(b)


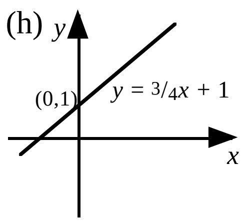
(c)


(d)


(e)


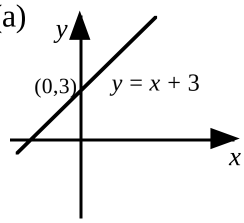
(f)


(g)


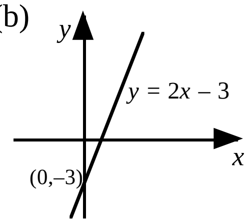
(h)


3. Line A – $a_6 = 2$ Line B – $a_5 = -1$ Line C – $a_1 = \frac{1}{2}$
   Line D – $a_3 = -\frac{1}{2}$ Line E – $a_4 = 0$ Line F – $a_2 = -3$

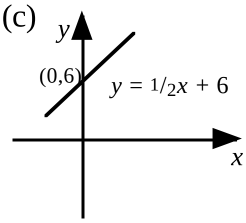
4. (a)



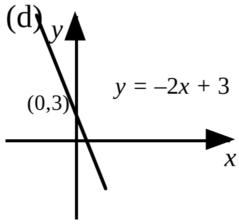
(b)



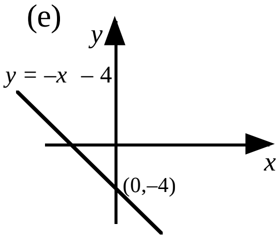
(c)



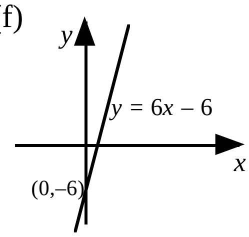
(d)



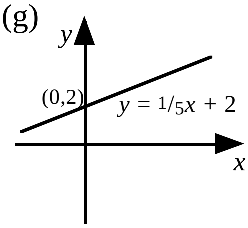
(e)



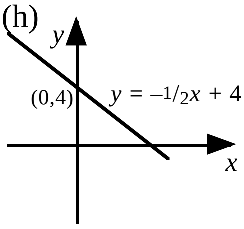
(f)



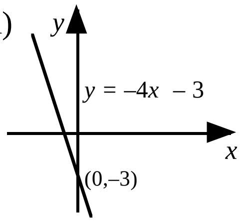
(g)



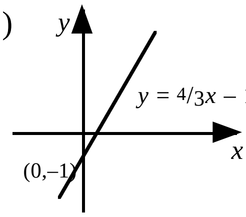
(h)



(i)



(j)



5. Line A – $y = 3x$ Line B – $y = –x + 6$ Line C – $y = x + 2$

Line D – $y = 2x – 1$ Line E – $y = 5x + 3$ Line F – $y = –{}^1/_4x + 1$

Line G – $y = {}^1/_2x – 4$ Line H – $y = –2x – 5$

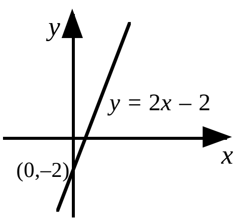
Exercise 3

1. Step 2: (0,–4); $y =$ a$x – 4$ Step 3: gradient = 3; $y = 3x – 4$
2. (a) $y = 2x + 1$ (b) $y = x + 3$ (c) $y = 4x – 3$
   (d) $y = {}^1/_2x + 2$ (e) $y = {}^1/_3x – 3$ (f) $y = 3x – 2$
3. (a) $y = –x + 2$ (b) $y = –3x – 4$ (c) $y = –2x + 2$
   (d) $y = –{}^1/_2x + 3$ (e) $y = –4x – 3$ (f) $y = –{}^5/_2x – 2$

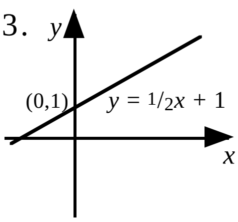
Check-up for Linear Relationships

1. (a) gradient = 4 (b) gradient = $–{}^3/_4$

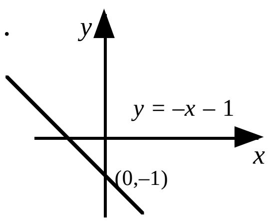
2.



3.



4.



5. $y = –3x – 2$
6. (a) $y = 3x – 4$ (b) $y = x + 2$ (c) $y = {}^1/_2x + 3$ (d) $y = –2x – 1$

Algebraic Operations

Exercise 1

1. (a) $6x + 12$ (b) $3a + 3$ (c) $5y - 20$ (d) $7t - 7$
   (e) $10x - 100$ (f) $4 + 2x$ (g) $12 + 3y$ (h) $30 - 6w$
   (i) $8 - 8c$ (j) $30 - 15h$ (k) $3x + 3y$ (l) $9a - 9c$
   (m) $8 - 4x$ (n) $11e - 11f$ (o) $1 - y$ (p) $y - 1$

2. (a) $6x + 12$ (b) $8a + 6$ (c) $5 + 10y$ (d) $18 - 18x$
   (e) $14w - 28$ (f) $cx + 5c$ (g) $dv + 3d$ (h) $gh - g$
   (i) $sr - 4s$ (j) $mn + 10m$ (k) $xv + xw$ (l) $ax + ar$
   (m) $xa - xy$ (n) $a^2 + ab$ (o) $r^2 - rs$ (p) $r^2 - r$
   (q) $a - a^2$ (r) $x^2 - 8x$ (s) $x^2 + 3xy$ (t) $3w^2 - w$
   (u) $5x^2 - 3x$ (v) $7ax - 5a^2$ (w) $4m^2 + 8mn$ (x) $27v - 2v^2$

3. (a) $2x + 2y + 8$ (b) $7x + 7y + 7$ (c) $5x - 5y - 30$ (d) $6x + 12y + 30$
   (e) $40x - 10y + 10z$ (f) $54a - 18b + 9$ (g) $3x^2 + 5xy + xz$ (h) $6a^2 - 8ab + 2ac$
   (i) $s^3 + 3s$ (j) $x^3 + x$ (k) $y^3 - y$ (l) $c^3 - 6c$
   (m) $w^3 + w^2$ (n) $a^3 - a^2$ (o) $x^4 - 2x^3$

Exercise 2A

1. (a) $x^2 + 6x + 5$ (b) $x^2 + 5x + 6$ (c) $x^2 + 11x + 30$ (d) $x^2 + 10x + 21$
   (e) $x^2 + 8x + 16$ (f) $x^2 + 2x + 1$ (g) $a^2 + 9a + 8$ (h) $s^2 + 21s + 110$
   (i) $w^2 + 104w + 400$

2. (a) $x^2 - 4x + 3$ (b) $x^2 - 6x + 8$ (c) $x^2 - 15x + 56$ (d) $a^2 - 7a + 10$
   (e) $b^2 - 14b + 49$ (f) $c^2 - 5c + 6$ (g) $v^2 - 20v + 100$ (h) $w^2 - 9w + 18$
   (i) $z^2 - 2z + 1$

3. (a) $x^2 + 6x + 5$ (b) $c^2 - 6c + 8$ (c) $s^2 - 3s - 18$ (d) $a^2 - 12a + 35$
   (e) $v^2 + 18v + 81$ (f) $q^2 - 4q - 12$ (g) $r^2 + 4r - 12$ (h) $w^2 - 64$
   (i) $x^2 - 1$ (j) $d^2 - 6d + 9$ (k) $a^2 + 5a - 66$ (l) $z^2 + z - 110$

4. (a) $4x^2 - 9$ (b) $25c^2 - 1$ (c) $4s^2 + 4s - 3$ (d) $4a^2 - 8a + 3$
   (e) $4v^2 + v - 3$ (f) $6q^2 + q - 12$ (g) $20r^2 + 2r - 6$ (h) $8w^2 + 10w - 25$
   (i) $100x^2 - 1$ (j) $2 - 3d + d^2$ (k) $12 + 5p - 2p^2$ (l) $1 - 5p + 6p^2$

5. (a) $x^2 + 4x + 4$ (b) $y^2 + 8y + 16$ (c) $z^2 + 6z + 9$ (d) $t^2 + 20t + 100$
   (e) $x^2 - 2x + 1$ (f) $y^2 - 12y + 36$ (g) $z^2 - 4z + 4$ (h) $t^2 - 16t + 64$
   (i) $a^2 + 2ab + b^2$ (j) $g^2 + 2gh + h^2$ (k) $r^2 - 2rs + s^2$ (l) $e^2 - 2ef + f^2$
   (m) $9x^2 + 6x + 1$ (n) $16x^2 - 24x + 9$ (o) $x^2 + 6xy + 9y^2$ (p) $a^2 - 8ab + 16b^2$
   (q) $16a^2 + 8ab + b^2$ (r) $25c^2 + 10cd + d^2$ (s) $25p^2 + 20pq + 4q^2$ (t) $4x^2 - 12xy + 9y^2$

Exercise 2B

1. $x^3 + 4x^2 + 4x + 1$
2. $x^3 - 2x^2 - 7x + 2$
3. $w^3 - 2w^2 - 5w + 6$
4. $z^3 - 6z^2 + 4z + 1$
5. $2v^3 + 5v^2 + 7v + 10$
6. $5a^3 - 35a^2 + 30a + 100$
7. $m^3 + 6m^2 + 12m + 8$
8. $n^3 - 3n^2 + 3n - 1$
9. $x^2 + 2 + {}^{1}/_{x^2}$
10. $x^2 - 2 + {}^{1}/_{x^2}$

Exercise 3

1. (a) $4(a + b)$ (b) $7(v + w)$ (c) $3(x - y)$ (d) $6(c - d)$ (e) $2(r + 2s)$
(f) $3(3m - 4n)$ (g) $a(v + w)$ (h) $p(q - r)$ (i) $b(x + 1)$ (j) $a(x^2 + 1)$
(k) $x(x + d)$ (l) $y(y - z)$ (m) $a(a + 1)$ (n) $t(t - 1)$ (o) $h^2(h + 1)$
(p) $m^2(m - 1)$ (q) $b(a + t)$ (r) $n(m - r)$ (s) $4(2x + 3y)$ (t) $7(5p - 3q)$
(u) $2a(a + 4b)$ (v) $3a(4b - 3c)$ (w) $pq(r + s)$ (x) $2c(4c - 1)$

2. (a) $m(a - b)$ (b) $5(4 - w)$ (c) $d(1 - d)$ (d) $z(y + 1)$ (e) $p(r - u)$
(f) $m(2n + p)$ (g) $2c(3d - 2e)$ (h) $3p(3q - 4r)$ (i) $2a(4a + 3)$ (j) $3x(5x - 2y)$
(k) $^1/_2(x + y)$ (l) $q(p + {}^1/_2sq)$ (m) $2ab(5a + 4b)$ (n) $^1/_2(1 + x)$ (o) $^1/_2(v - 3)$
(p) $2\pi r(h + r)$ (q) $3(2a + b - 4c)$ (r) $m(n - p + m)$ (s) $x(3x - 2y + 6)$ (t) $5x^2(5 - y)$

Exercise 4

1. (a) $(x - y)(x + y)$ (b) $(p - q)(p + q)$ (c) $(d - e)(d + e)$ (d) $(x - 3)(x + 3)$
(e) $(y - 4)(y + 4)$ (f) $(t - 5)(t + 5)$ (g) $(5 - t)(5 + t)$ (h) $(9 - q)(9 + q)$
(i) $(1 - v)(1 + v)$ (j) $(x - 2)(x + 2)$ (k) $(k - 5)(k + 5)$ (l) $(n - 6)(n + 6)$
(m) $(d - 10)(d + 10)$ (n) $(e - 11)(e + 11)$ (o) $(12 - y)(12 + y)$ (p) $(7 - x)(7 + x)$
(q) $(x - 1)(x + 1)$ (r) $(1 - y)(1 + y)$ (s) $(9 - a)(9 + a)$ (t) $(100 - b)(100 + b)$

2. (a) $(3a - 2)(3a + 2)$ (b) $(2b - 5)(2b + 5)$ (c) $(4c - 1)(4c + 1)$ (d) $(5d - 6)(5d + 6)$
(e) $(3e - 4)(3e + 4)$ (f) $(5f - 9)(5f + 9)$ (g) $(2g - h)(2g + h)$ (h) $(j - 5k)(j + 5k)$
(i) $(8m - 7n)(8m + 7n)$ (j) $(2p - 3q)(2p + 3q)$ (k) $(9r - 1)(9r + 1)$ (l) $(1 - 8s)(1 + 8s)$
(m) $(11 - 4t)(11 + 4t)$ (n) $(10u - 11v)(10u + 11v)$
(o) $(100\text{w} - 1)(100\text{w} + 1)$ (p) $(5x - 7y)(5x + 7y)$

3. (a) $2(a - 3)(a + 3)$ (b) $5(b - 1)(b + 1)$ (c) $6(c - 3)(c + 3)$ (d) $4(d - 2)(d + 2)$
(e) $7(e - g)(e + g)$ (f) $6(p - 2q)(p + 2q)$ (g) $10(x - 3y)(x + 3y)$ (h) $12(u - v)(u + v)$
(i) $a(m - n)(m + n)$ (j) $k(a - 5b)(a + 5b)$ (k) $n(r - 9q)(r + 9q)$ (l) $d(d - 7)(d + 7)$
(m) $b(8 - b)(8 + b)$ (n) $2u(u - 4)(u + 4)$
(o) $3w(2w - 3)(2w + 3)$ (p) $11x^3(x - 1)(x + 1)$

Exercise 5

1. $(x + 2)(x + 1)$
2. $(x + 3)(x + 2)$
3. $(x + 1)(x + 1)$
4. $(y + 5)(y + 1)$
5. $(y + 10)(y + 1)$
6. $(y + 7)(y + 1)$
7. $(v + 4)(v + 5)$
8. $(v + 2)(v + 5)$
9. $(v + 4)(v + 2)$
10. $(w - 1)(w - 1)$
11. $(w - 2)(w - 2)$
12. $(w - 3)(w - 3)$
13. $(a - 2)(a - 1)$
14. $(a - 3)(a - 4)$
15. $(a - 7)(a - 1)$
16. $(c - 6)(c - 7)$
17. $(c - 8)(c - 3)$
18. $(c - 1)(c - 9)$
19. $(s + 6)(s + 6)$
20. $(s - 6)(s - 6)$
21. $(s + 7)(s + 7)$
22. $(z - 7)(z - 7)$
23. $(z + 4)(z + 9)$
24. $(z - 4)(z - 9)$
25. $(b + 36)(b + 1)$
26. $(b - 36)(b - 1)$
27. $(b - 9)(b - 9)$
28. $(p + 3)(p + 3)$
29. $(p - 8)(p + 1)$
30. $(p + 2)(p + 2)$
31. $(m + 5)(m + 6)$
32. $(m + 4)(m - 3)$
33. $(m + 2)(m - 3)$
34. $(n - 5)(n - 3)$
35. $(n - 2)(n + 5)$
36. $(n + 1)(n - 4)$
37. $(r - 4)(r + 2)$
38. $(r - 1)(r + 6)$
39. $(r + 6)(r + 6)$
40. $(e - 7)(e + 2)$
41. $(e + 3)(e + 4)$
42. $(e - 8)(e + 7)$
43. $(g - 4)(g - 3)$
44. $(g + 2)(g - 3)$
45. $(g - 4)(g + 3)$
46. $(k + 1)(k - 5)$
47. $(k + 3)(k - 2)$
48. $(k + 7)(k - 5)$
49. $(y + 6)(y - 2)$
50. $(y + 6)(y - 3)$
51. $(y + 4)(y - 7)$
52. $(x + 5)(x - 8)$
53. $(x + 3)(x - 5)$
54. $(x + 5)(x + 6)$
55. $(v - 1)(v - 8)$
56. $(v - 3)(v + 8)$
57. $(v + 3)(v - 8)$
58. $(w + 6)(w - 4)$
59. $(w - 6)(w + 4)$
60. $(w + 12)(w - 2)$
61. $(a + 2)(a - 12)$
62. $(a + 24)(a - 1)$
63. $(a + 1)(a - 24)$
64. $(b + 10)(b - 3)$
65. $(b + 5)(b - 9)$
66. $(b + 2)(b - 9)$

67. $(c + 7)(c + 8)$
68. $(c - 9)(c - 6)$
69. $(c + 9)(c + 9)$
70. $(d + 2)(d - 14)$
71. $(d + 50)(d - 1)$
72. $(d - 1)(d - 50)$
73. $(a + b)(a + b)$
74. $(x - y)(x - y)$
75. $(p - 2q)(p + q)$

Exercise 6

1. $(x + 3)(2x + 1)$
2. $(2y + 3)(y + 1)$
3. $(3w + 1)(w + 2)$
4. $(2a + 3)(5a + 1)$
5. $(2b + 1)(3b + 2)$
6. $(6c + 1)(c + 1)$
7. $(3d + 5)(d + 3)$
8. $(2m + 3)(5m + 2)$
9. $(2p - 1)(p - 3)$
10. $(2n - 1)(6n - 1)$
11. $(2q - 1)(q - 3)$
12. $(2x - 3)(3x - 2)$
13. $(4s - 5)(2s - 1)$
14. $(3r - 4)^2$
15. $(3g - 2)(4g - 5)$
16. $(3k - 2)(k - 1)$
17. $(3y + 4)(y - 2)$
18. $(3w + 1)(w - 2)$
19. $(2u - 3)(3u + 2)$
20. $(5v - 1)(v + 1)$
21. $(2x - 1)(x + 1)$
22. $(3d + 1)(d - 1)$
23. $(4a + 3)(2a - 1)$
24. $(4y - 5)(3y + 1)$
25. $(4p - 3)(p - 2)$
26. $(3 - 2x)(5 + x)$
27. $(5 - 4x)(1 + 3x)$
28. $(1 - 4x)^2$
29. $(1 - 6x)(1 + 3x)$
30. $(4p + q)(p - 2q)$

Exercise 7

1. $4(x + 3y)$
2. $(a - 9)(a + 9)$
3. $(w + 5)^2$
4. $y(y - 1)$
5. $(v - 4)(v + 3)$
6. $(1 - b)(1 + b)$
7. $(u + 6)^2$
8. $a(p - q + r)$
9. $7(x - 2)(x + 2)$
10. $(w - r)(w + r)$
11. $h(h - 11)$
12. $(x - 1)^2$
13. $(t + 1)(t - 1)$
14. $t(t - 1)$
15. $(a - 3)(a + 1)$
16. $3(c - 4)(c + 4)$
17. $5d(d - 4)$
18. $a^3(a - 1)$
19. $(2s + 5)(s - 1)$
20. $(x - 6)^2$
21. $(4y + 1)^2$
22. $(7 - g)(7 + g)$
23. $4(3 - r)(3 + r)$
24. $7z(2 - z)$
25. $(5 - 3g)(5 + 3g)$
26. $(2b + 1)(b - 1)$
27. $(2x + 3)(3x - 1)$
28. $11(u - 2v)(u + 2v)$
29. $7(3u^2 + 4v^2)$
30. $(5p - 1)^2$
31. $3mn(m - 2n)$
32. $(1 - n^2)$
33. $(3 - s)(9 + s)$
34. $3a(a - 4)(a + 4)$
35. $2(2n - 1)(2n + 3)$
36. $2(2n - 1)^2$
37. $5(r - 1)(r + 2)$
38. $2(2w - 1)(w + 4)$
39. $7x(1 - 3x)(1 + 3x)$
40. $9x(1 + 3x^2)$
41. $x(y - z)(y + z)$
42. $(2e + 3)(e - 7)$
43. $(x - 1)(x + 1)(x^2 + 1)$
44. $2(1 - q)^2$
45. $(g + 3h)(g - 2h)$
46. $(2k + \pi r)(k + \pi r)$
47. $a^2(1 - a)(1 + a)(1 + a^2)$
48. $(k^2 + 1)^2$
49. $2(a^2 + 2)(a^2 - 3)$
50. $b(b - 3)(b + 3)(b^2 + 9)$
51. $(3x^2 - 1)(x^2 + 2)$
52. $(3x^2 - 4)^2$
53. $(2x^2 - 3)(x^2 + 1)$
54. $(1 - y)(1 + y)(1 + y^2)(1 + y^4)$

Checkup for Algebraic Operations

1. (a) $12x + 3$ (b) $ya - y^2$ (c) $v^2 - v$ (d) $14w^2 - 35w$
   (e) $18x + 12y - 6$ (f) $c^3 + c^2 - c$ (g) $12da + 9db$ (h) $gh^2 - g^3$
   (i) $18x^2 + 12xy - 6x$ (j) $c^4 + c^3 - 4c^2$ (k) $3a^2b + 4ab^2$ (l) $10pq - 2pq^2$

2. (a) $x^2 + 8x + 7$ (b) $x^2 - 5x + 6$ (c) $x^2 - x - 30$ (d) $x^2 + 6x - 27$
   (e) $x^2 + 2x + 1$ (f) $x^2 - 4x + 4$ (g) $20x^2 + 31x - 7$ (h) $12x^2 - 12x + 3$
   (i) $6x^2 - 10x - 4$ (j) $4x^2 - 12x + 9$ (k) $4x^3 - 11x^2 + 8x - 4$
   (l) $x^3 - 9x^2 + 27x - 27$

3. (a) $9(m - n)$ (b) $3(2a - 5b)$ (c) $y(1 - y)$
   (d) $2(7p^2 + 3q)$ (e) $p(3r + u)$ (f) $2p(2p + 3q - 1)$
   (g) $3(2x + 10y - 5z)$ (h) $3p(3q - 4r)$ (i) $(r - s)(r + s)$
   (j) $(9 - q)(9 + q)$ (k) $(4r - 7)(4r + 7)$ (l) $2(b - 4)(b + 4)$

(m) $5w(2w-3)(2w+3)$ (n) $(y-2)(y-1)$ (o) $(a-10)(a+3)$
(p) $(y-2)(y+3)$ (q) $(12-r)(2+r)$ (r) $(x-7)^2$
(s) $(2p-3)(3p-4)$ (t) $(2x+1)^2$ (u) $2(q+8)(q-9)$
(v) $(2x-y)(x+2y)$ (w) $2(a^2+1)(3a^2-2)$ (x) $(5y^2+3)(y^2-3)$

Properties of the Circle

Exercise 1

1. (i) 5·2 cm (ii) 9·42 cm (iii) 25·1 cm (iv) 14·0 cm (v) 47·1 cm (vi) 44·7 cm (vii) 42·4 cm
2. 47·1 cm 3. 83·7 cm 4. 314 m 5. 45° 33·0 inches 6. 92°

Exercise 2

1. (a) 105 cm^2 (b) 177 cm^2 (c) 471 cm^2 (d) 236 cm^2 (e) 367 cm^2 (f) 377 cm^2
2. 2152 cm^2 3. 1·64 m^2 4. 2261 cm^2
5. (a) 112 cm^2 (b) 134 cm^3 6. (a) 179 cm^2 (b) 2144 cm^3 7. 90°

Exercise 3

1. $a = 38$, $b = 25$, $c = 30$, $d = 20$, $e = 125$, $f = 20$, $g = 130$, $h = 70$, $i = 24$, $j = 26$, $k = 51$.
2. $w = 117$ 3. $x = 63$ 4. $v = 12$, $w = 7$
5. $x = 30$, $y = 38·9$, $z = 66$ 6. (a) 100mm (b) $p = 48$, $q = 42$, $r = 42$

Exercise 4

1. $a = 59$, $b = 45$, $c = 15$, $d = 50$, $e = 34$, $f = 40$, $g = 59$.
2. (a) 23° (b) 49° 3. (a) 12·2 (b) 12·0
4. (a) $v = 100$, $w = 36·9$ (b) $v = 10·5$, $w = 58·2$ (c) $v = 10·2$, $w = 42·8$ (d) $v = 96·6$, $w = 75$

Exercise 5

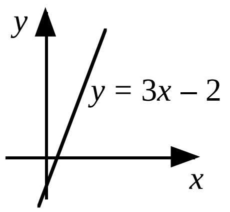
1. (a) 140° 20° 20° (b) 20° 20° 70° 70° (c) 10° 10° 80° 80° 90° 90° 90° 90° (d) 4 x 90°, 4 x 75°, 4 x 15°
2. (a) 6 (b) 7 (c) 4·77 3. (a) 11·0 (b) 14·4
4. $a = 7·8$, $b = 8·9$, $c = 41·4$, $d = 48·6$.
5. 18 cm 6. 73·9 cm 7. 59·2 cm

Checkup for Properties of the Circle

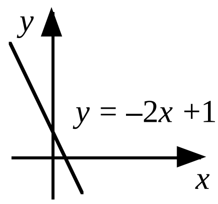
1. (a) 230 mm (b) 26·2 cm (c) 126 m
2. (a) 245 cm^2 (b) 698 cm^2 (c) 961 cm^2
3. (a) 15·7 cm (b) 70·7 cm^2 4. 6978 mm^2
5. a = 115, b = 140, c = 52.
6. a = 68·2, b = 24, c = 42, d = 190, e = 72
7. 18 cm

Specimen Assessment Questions

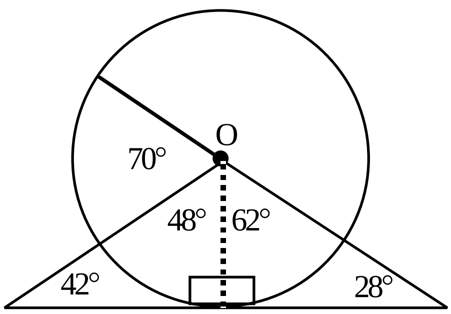
1. £556·92 2. £500 3. 9280000 pesetas 4. 17%
5. (a) 385 cm^3 (b) 308 cm^3 (c) 558·1 cm^3
6. (a) 2000·18 cm^3 (b) 615·44 cm^3 (c) 11·5 cm^3
7. 310·86 cm^3
8. $m_{AB} = {}^1/_2$ $m_{PQ} = -7$
9. (a)

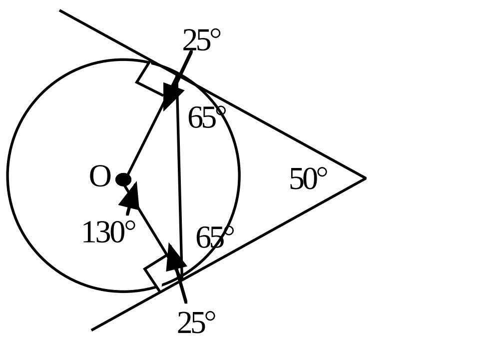


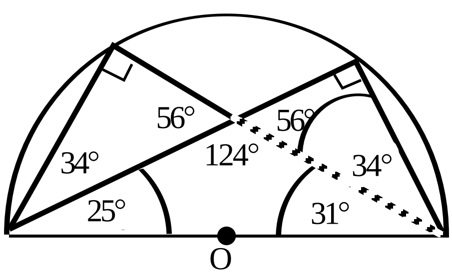
(b)

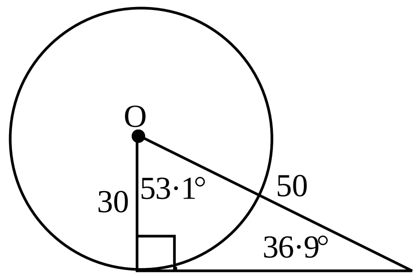


10. (a) $y = 1/2x + 1$ (b) $y = -x - 2$
11. (a) $21x - 14$ (b) $6v - 27v^2$ (c) $6x - 30y + 12$
 (d) $2x^3 - 2x^2 + 2x$ (e) $42pq + 12p^2$ (f) $6x^3y - 2x^2y^2$
 (g) $2k^2 + k - 21$ (h) $9a^2 - 30a + 25$ (i) $3x^3 + 13x^2 - 13x + 2$
12. (a) $3(3a - 7b)$ (b) $v(x + y)$ (c) $\pi m(m - 2)$
 (d) $(2d - 3e)(2d + 3e)$ (e) $2(x - 9)(x + 9)$ (f) $t(1 - t)(1 + t)$
 (g) $(x + 3)(x + 4)$ (h) $(a - 3)(a - 6)$ (i) $(2y + 1)(3y - 4)$
 (j) $2(b - 5)(b + 2)$ (k) $(4p + q)(p - 3q)$ (l) $(9 - w)(2 + w)$
13. (i) (a) 6·98 cm (b) 27·2 cm (ii) (a) 17·4 cm^2 (b) 163 cm^2
14. (a) 22·7 cm (b) 25·1 cm^2
15.









16. (a) 30 (b) 25 (c) 0·8 17. 36 cm